

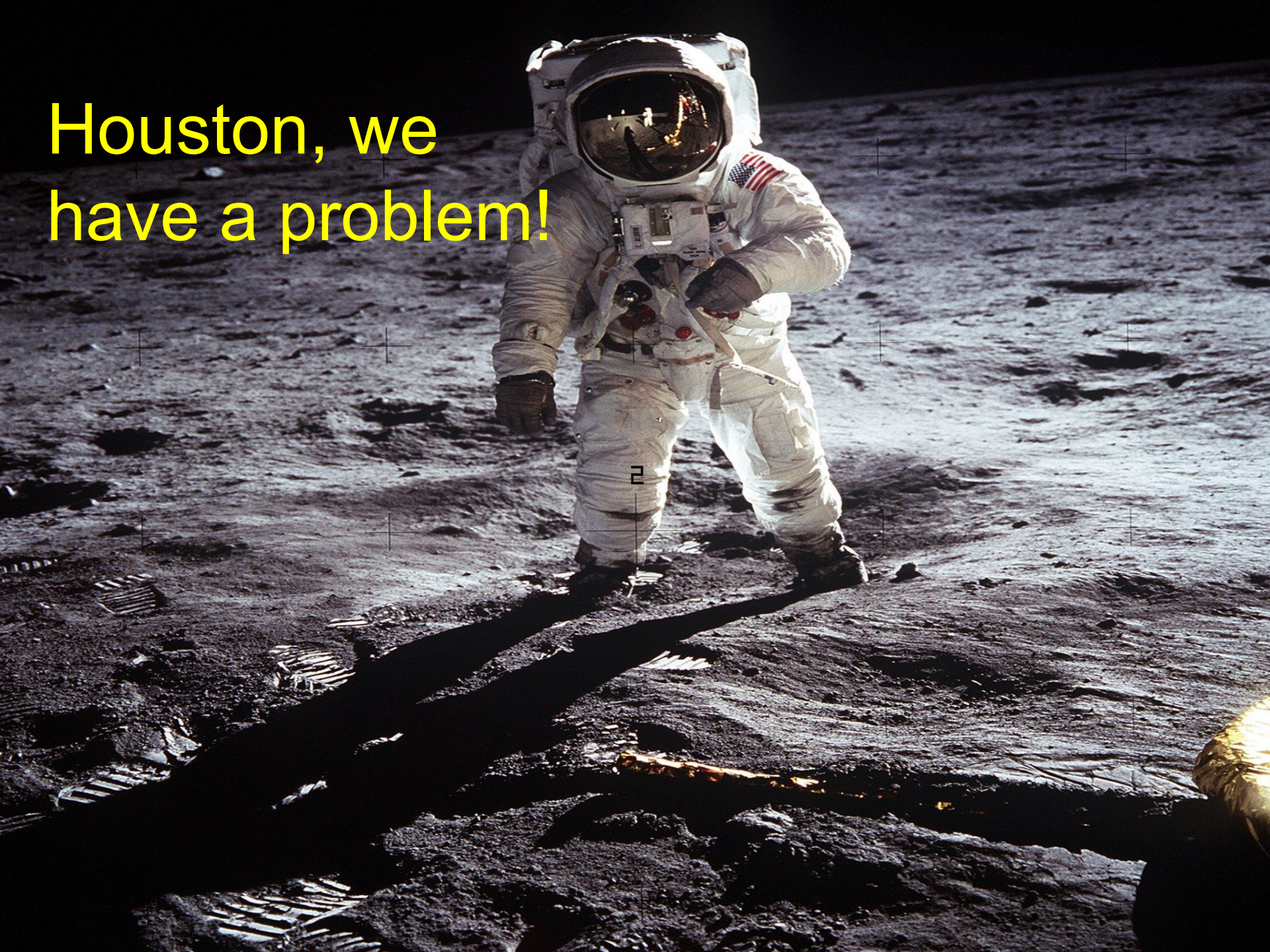
# Anonymous Credentials

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Houston, we  
have a problem!





A full-page photograph of astronaut Buzz Aldrin on the Moon. He is wearing a white spacesuit with an American flag patch on the right shoulder and is standing on the dark, cratered lunar surface. His shadow is cast long and dark to his left. The background shows the horizon of the Moon under a black sky.

Houston, we  
have a problem!

“Buzz Aldrin's footprints are still up there”  
(Robin Wilton)





- Data storage ever cheaper → “store by default”
  - also collateral collection, surveillance cameras, Google Street View with wireless traffic, Apple location history,...
- Data mining ever better
  - self-training algorithms cleverer than their designers
  - not just trend detection, even prediction, e.g., flu pandemics, ad clicks, purchases,...
  - what about health insurance, criminal behavior?

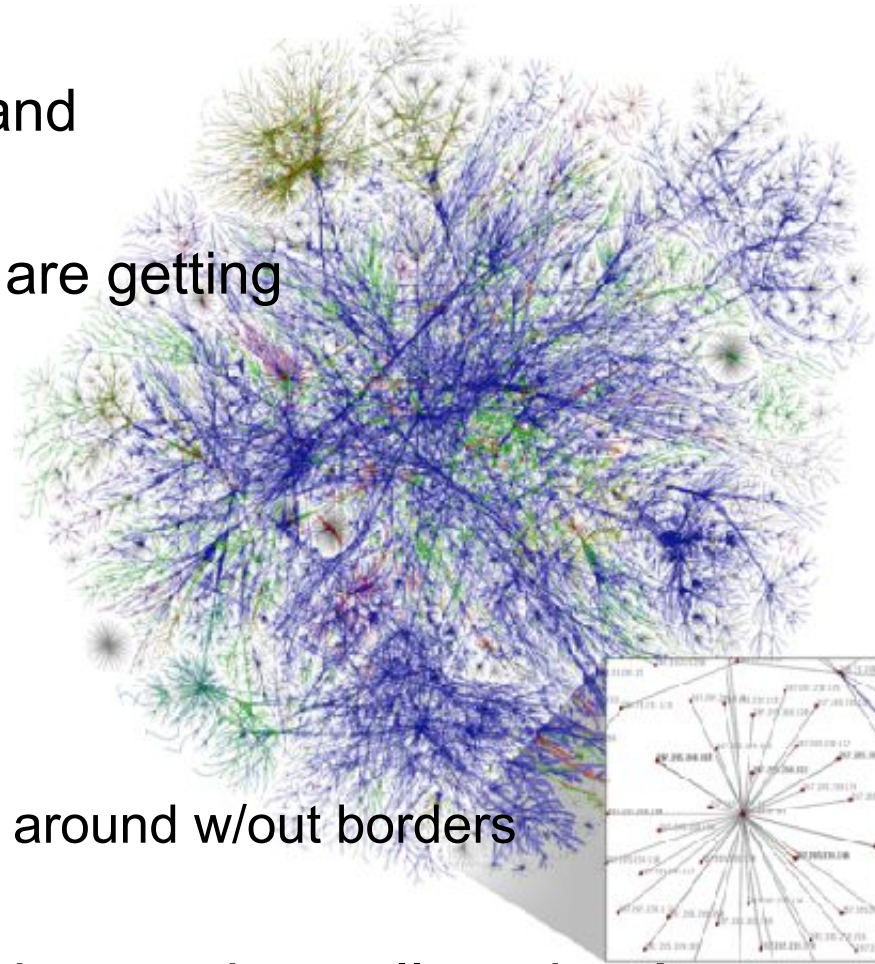


- The world as we know it
  - Humans forget most things too quickly
  - Paper collects dust in drawers

*We build apps with the paper-based world in mind :-(  
– if it works it works  
– security too often still an afterthought  
– implementors too often have no crypto education*

The ways of data are hard to understand

- Devices, operating systems, & apps are getting more complex and intertwined
  - Mashups, Ad networks
  - Not visible to users, and experts
  - Data processing changes constantly
- And the cloud makes it worse...
  - Processing machines can be moved around w/out borders



Far too easy to lose (control over) data and to collect data!

... “The NSA has all our data anyway”

... “I have nothing to hide!”

- Huge security problem!

- Millions of hacked passwords (100'000 followers \$115 - 2013)
- Stolen identities (\$150 - 2005, \$15 - 2009, \$5 – 2013)



- Difficult to put figures down

- Credit card fraud
- Spam & marketing
- Manipulating stock ratings, etc..
- (Industrial) espionage



- We know secret services can do it easily, but they are not the only ones

- but this is not about homeland security
- and there are limits to the degree of protection that one can achieve



- last but not least: data are the new money, so they need to be protected!

No, but we need paradigm shift &  
build stuff for the moon  
rather than the sandy beach!

- devices, sensors, etc cannot all be physically protected
  - authentication of all devices
  - authentication of all data
  - ...makes it even worse :-)
  
- data cannot be controlled
  - minimize information
  - encrypt information
  - attach usage policies to each bit



- Legal approach
  - Regulate what information can be collected
  - How to collect it
  - How to use and protect it
  - Issue fines for misbehavior
  - Very different for different countries and cultures
  
- Technological approach
  - Protect data by encryption
  - Govern data by policies
  - Minimize data that needs to be used

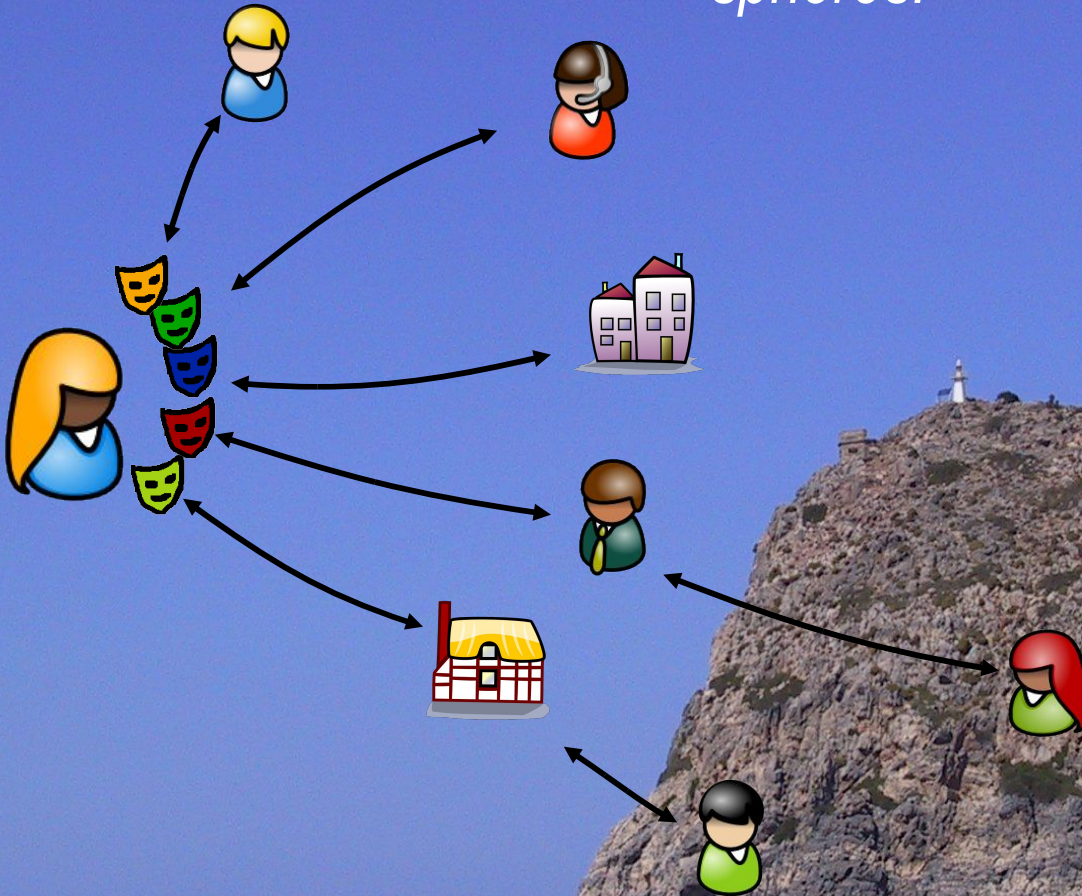
- tracing is so easy
  - each piece of hardware is quite unique
  - log files everywhere
  
- .... but that's not the point!
  - it's not about NSA et al.
  - active vs. passive “adversaries”

..... still, *privacy by design!*



# Our Vision

*In the Information Society, **users** can act and interact in a **safe and secure** way while **retaining** control of their private spheres.*



## Privacy, Identity, and Trust Mgmt Built-In Everywhere!

- Network Layer Anonymity
  - ... in mobile phone networks
  - ... in the Future Internet as currently discussed
  - ... access points for ID cards
  
- Identification Layer
  - Access control & authorization
  
- Application Layer
  - “Standard” e-Commerce
  - Specific Apps, e.g., eVoting, OT, PIR, .....
  - Web 2.0, e.g., Facebook, Twitter, Wikis, ....



A photograph of a beach at sunset or sunrise. The ocean waves are breaking on the shore, creating a white foam. The sky is a mix of orange, yellow, and blue. In the foreground, a single footprint is visible in the dark sand.

# Privacy at the Authentication Layer

## Authentication without identification



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# Let's see a scenario





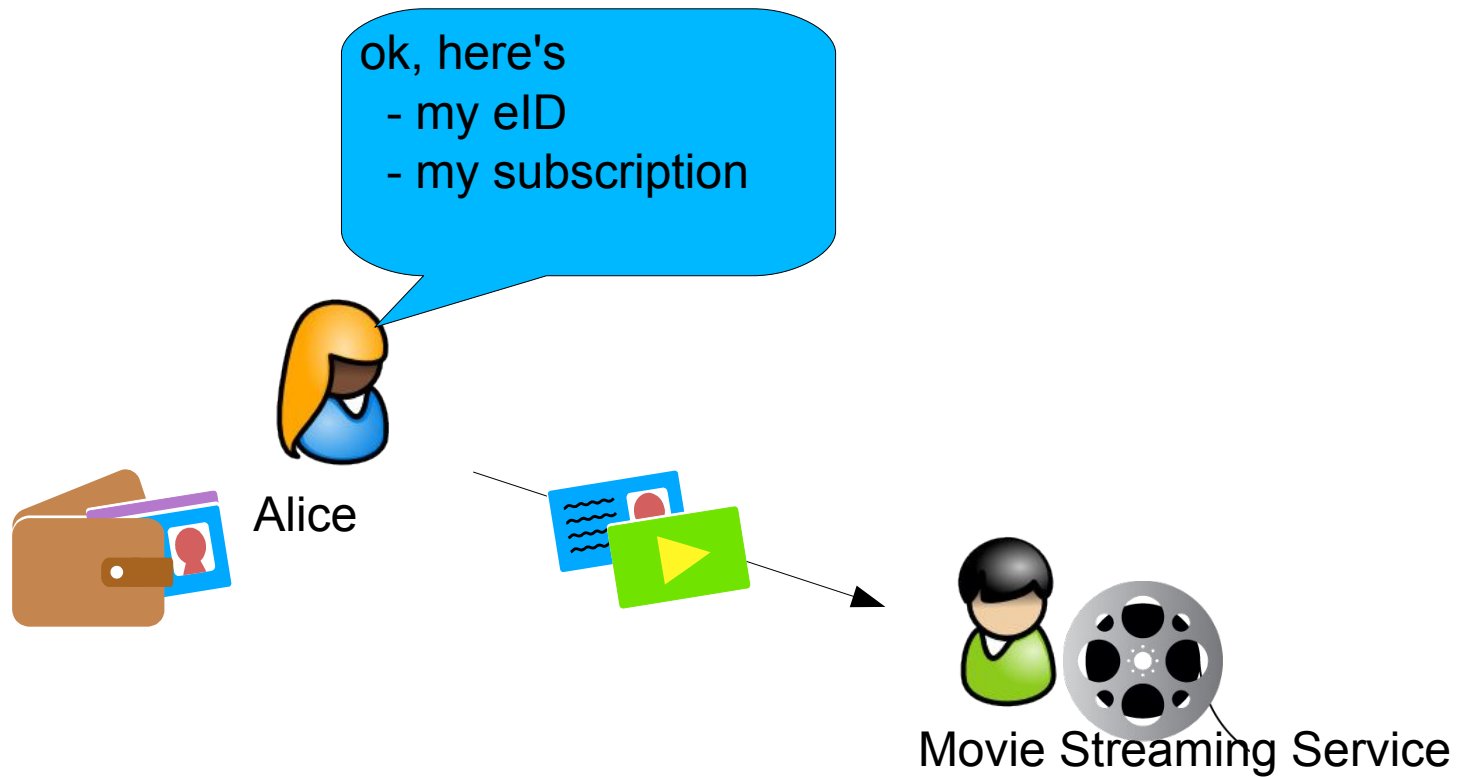
Alice

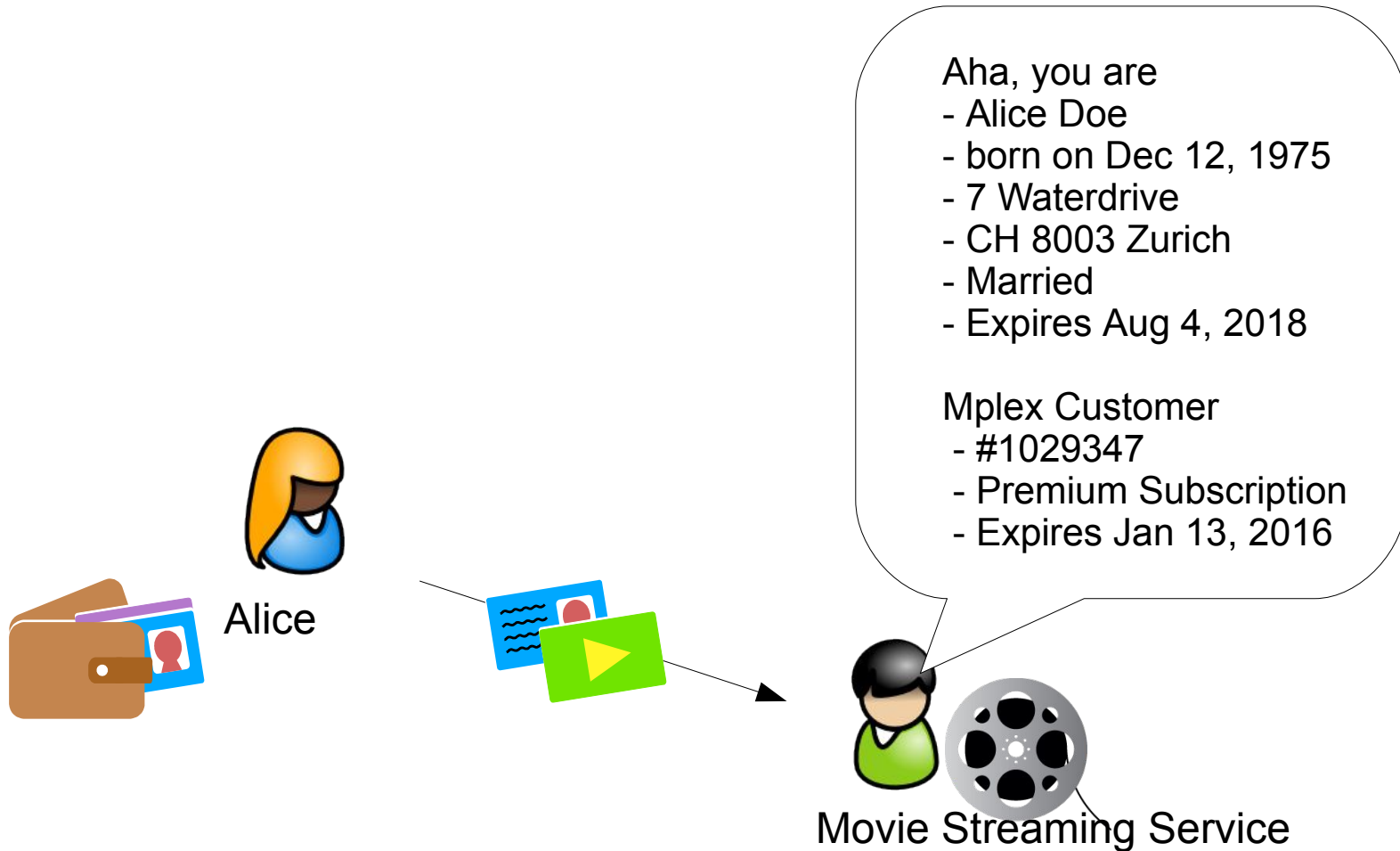
You need:  
- subscription  
- be older than 12



Movie Streaming Service

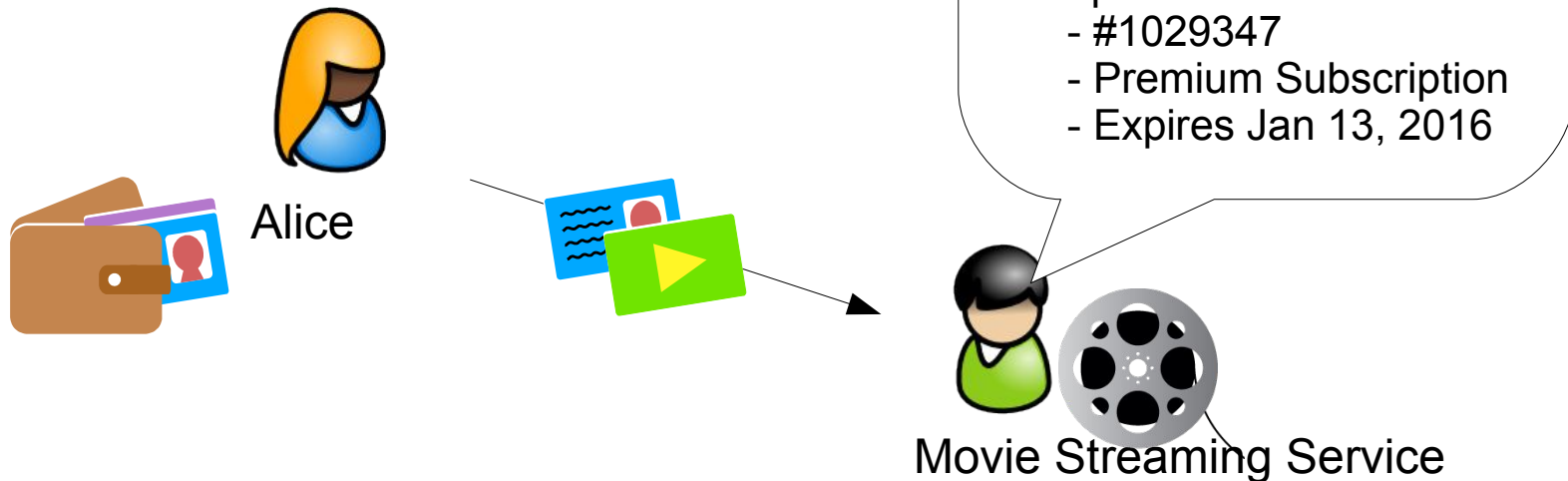






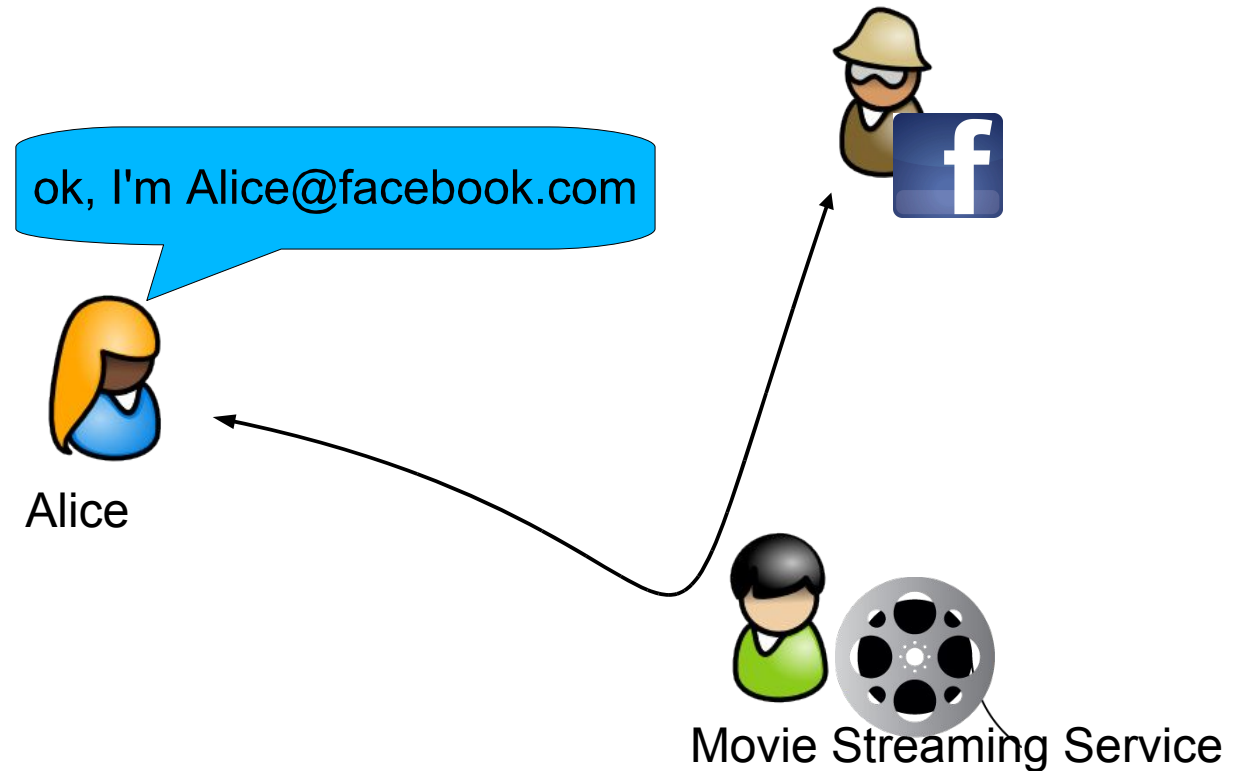
This is a privacy and security problem!

- - identity theft
- - profiling
- - discrimination

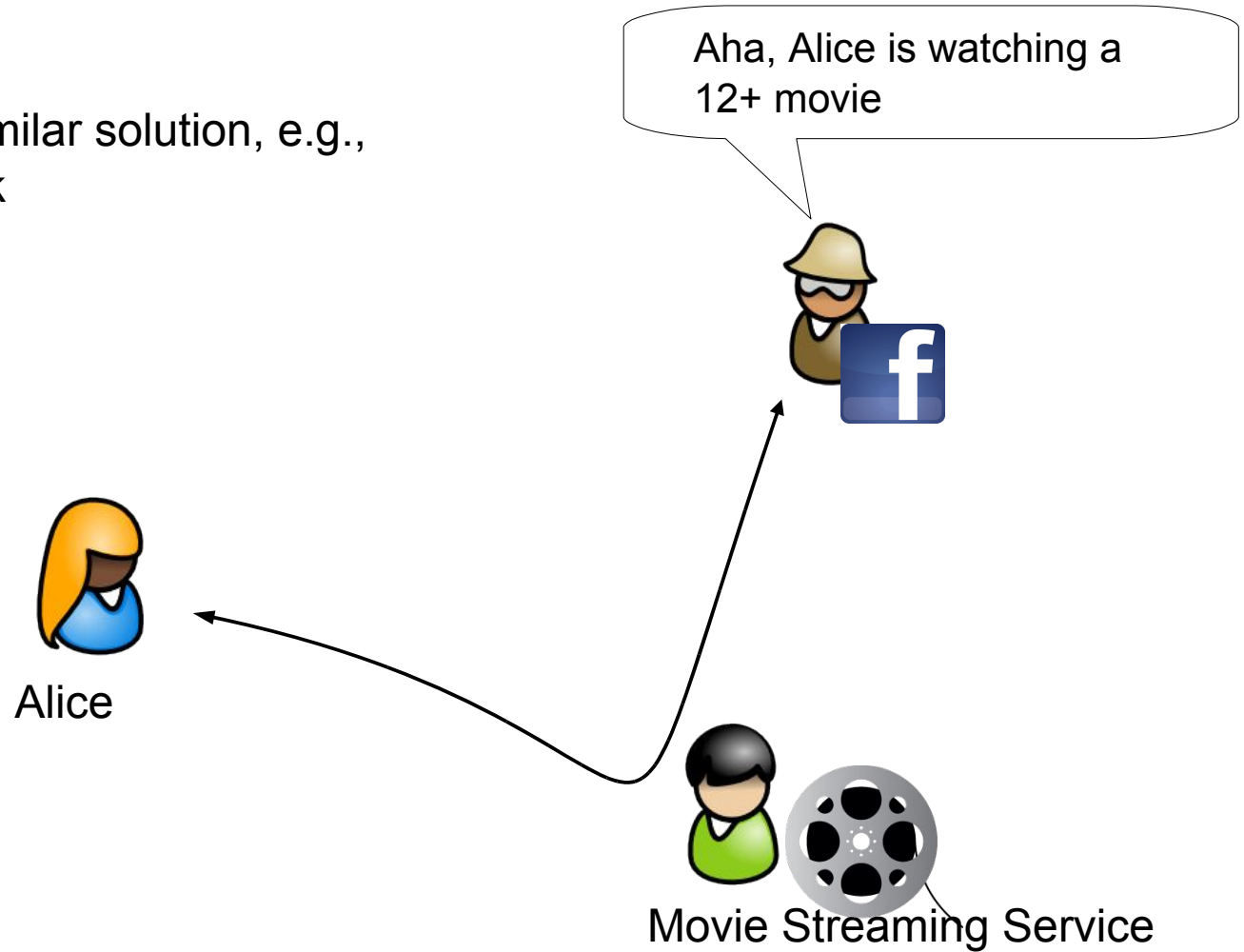




With OpenID and similar solution, e.g.,  
log-in with Facebook



With OpenID and similar solution, e.g.,  
log-in with Facebook



With OpenID and similar solution, e.g.,  
log-in with Facebook



Alice

Aha, Alice is watching a  
12+ movie

Aha, you are

- Alice@facebook.com
- born on Dec 12, 1975
- Alice's friends are ....
- Alice's public profile is ...

Mplex Customer

- #1029347
- Premium Subscription
- Expires Jan 13, 2016



Movie Streaming Service



Identity Mixer solves this.

When Alice authenticates to the Movie Streaming Service with Identity Mixer, all the services learns is that Alice

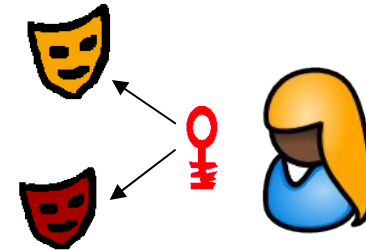
has a subscription

is older than 12

and no more.

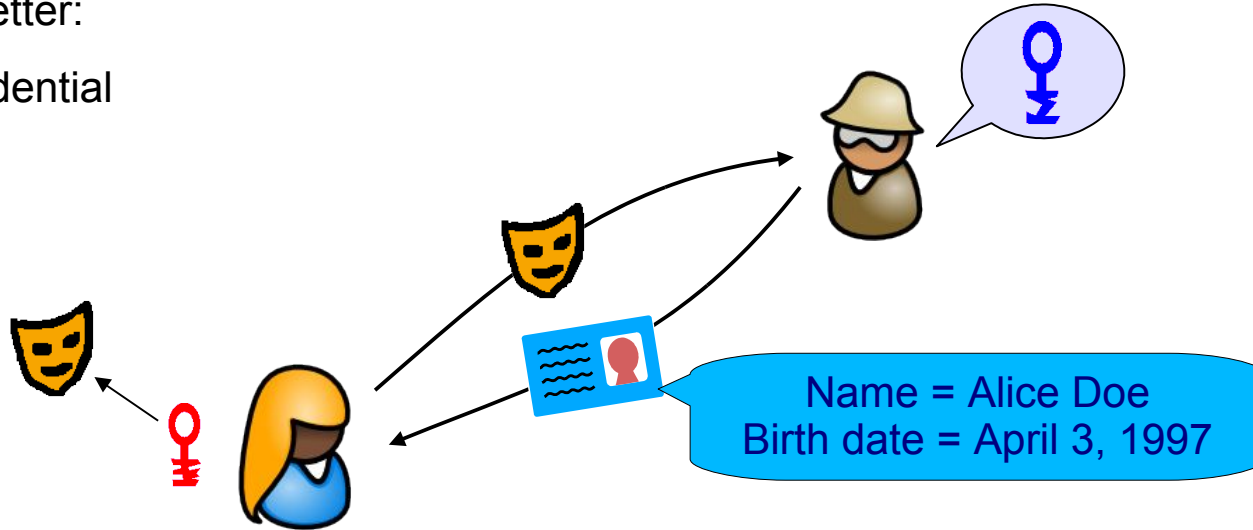
Like PKI, but better:

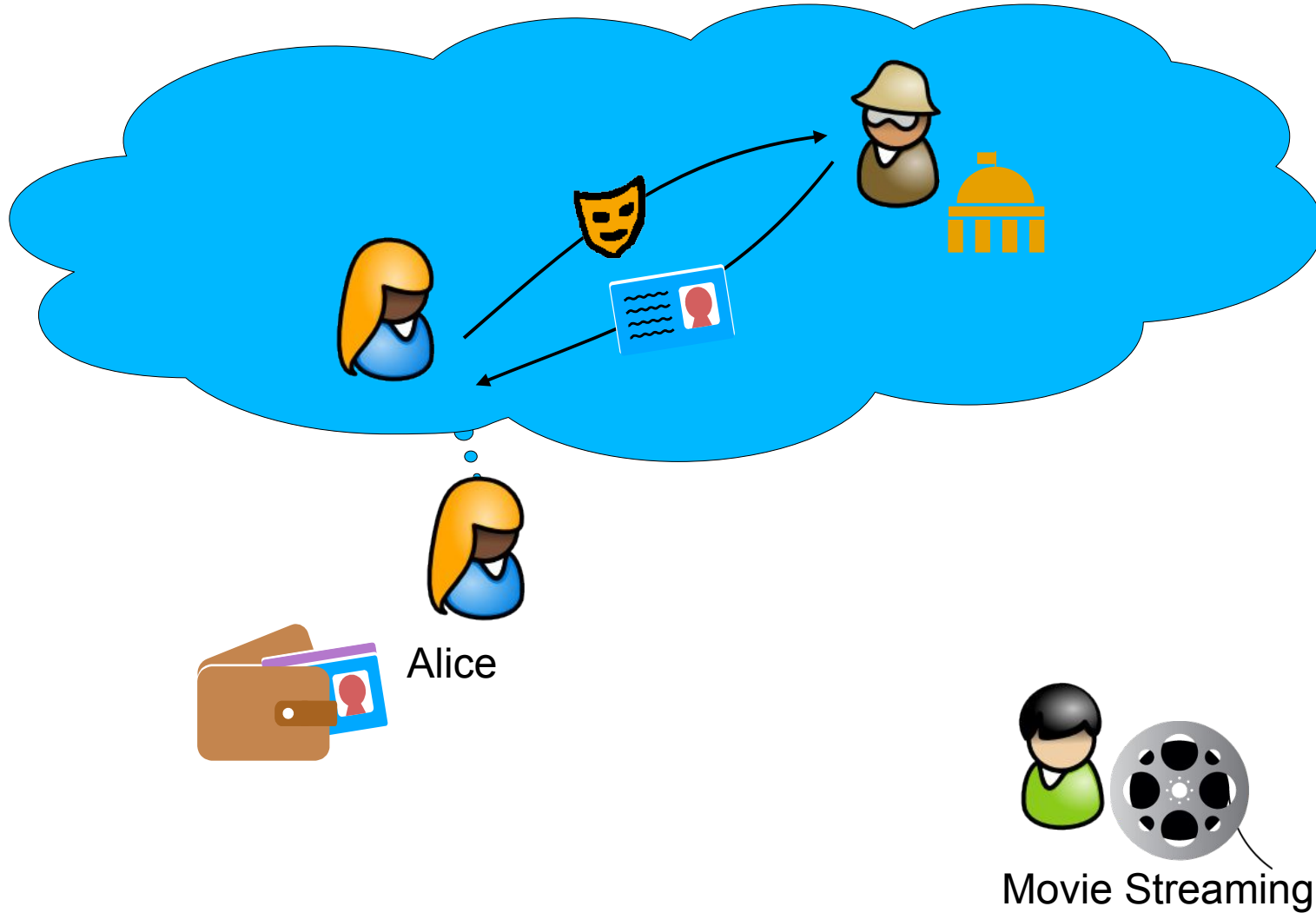
- One secret Identity (secret key)
- Many Public Pseudonyms (public keys)



Like PKI, but better:

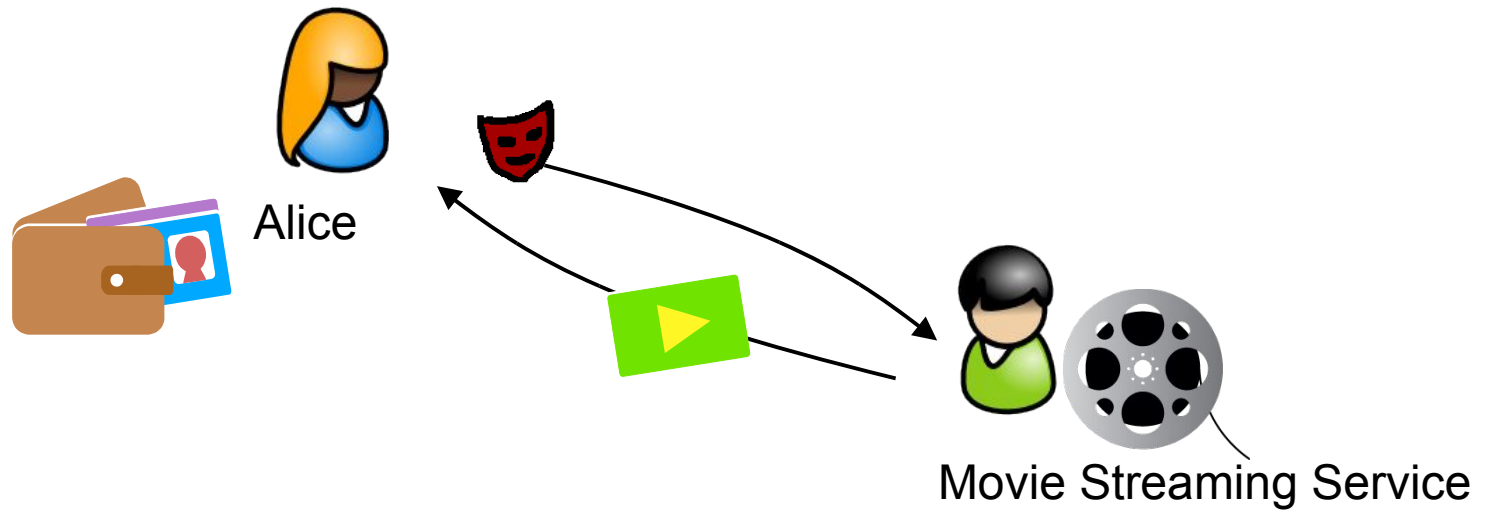
- Issuing a credential







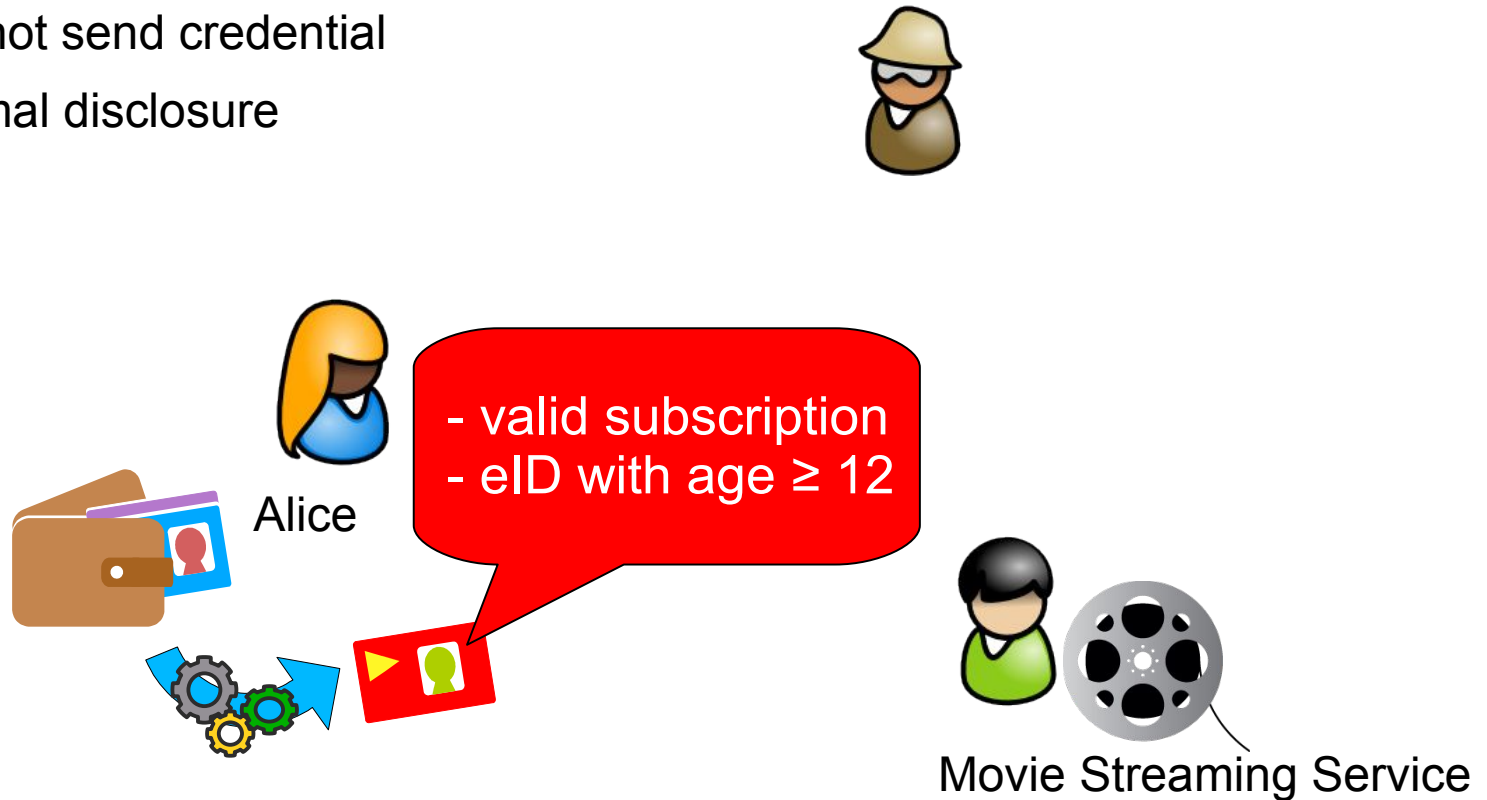






Like PKI

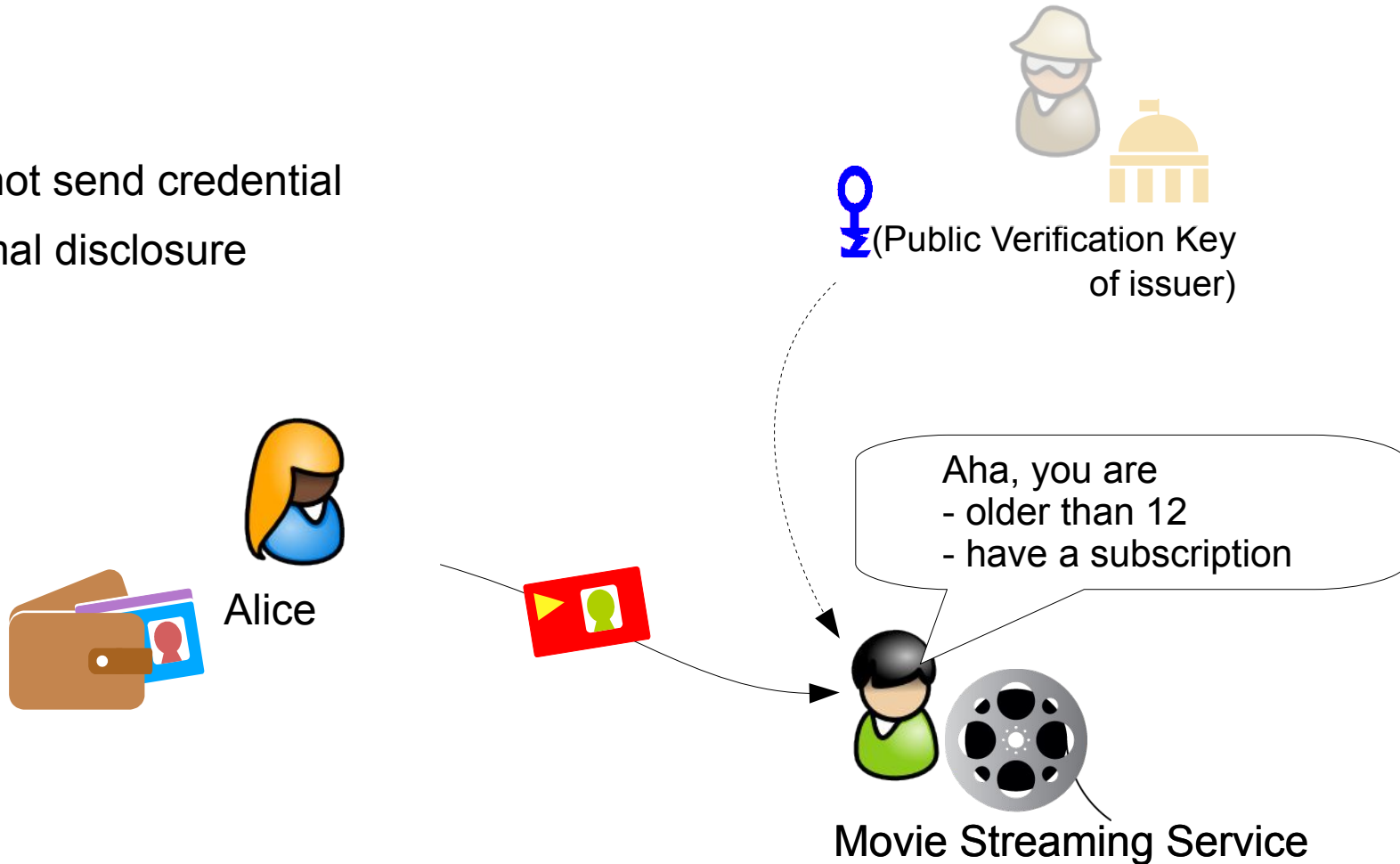
- but does not send credential
- only minimal disclosure





Like PKI

- but does not send credential
- only minimal disclosure

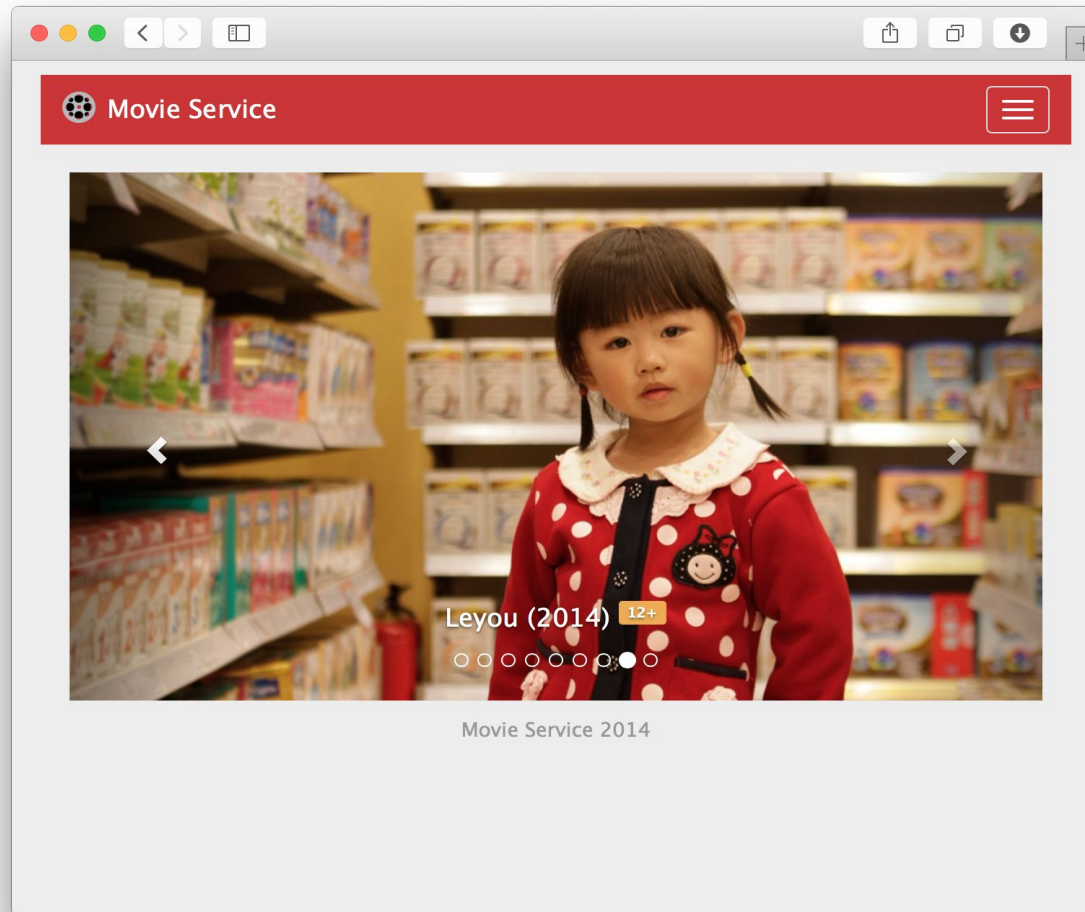


- **For Users: privacy**

- minimizing disclosure of personal data
- keeping their identities safe
- pseudonymous/anonymous access

- **For Service Providers: security, accountability, and compliance**

- avoiding the risk of losing personal data if it gets stolen
- compliance with legislation (access control rules, personal data protection)
- strong authentication (cryptographic proofs replace usernames/passwords)
- user identification if required (under certain circumstances)

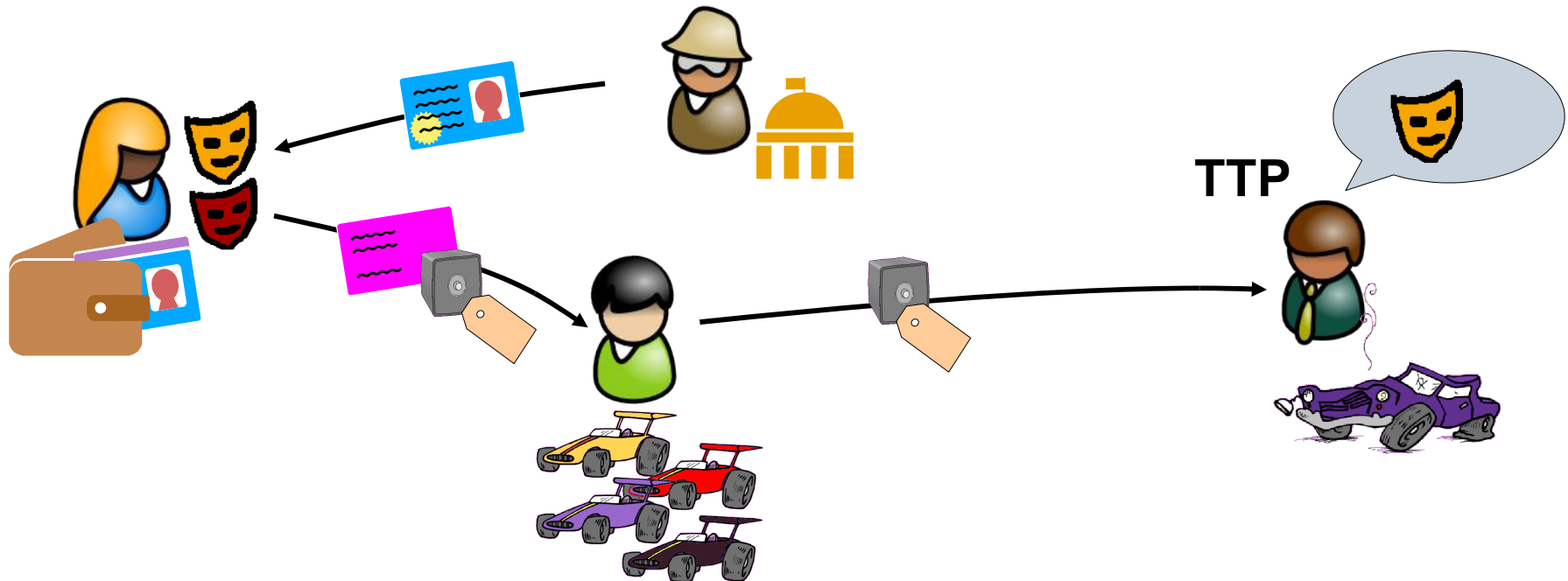


Try yourself at [www.ibm.biz/identitymixer](http://www.ibm.biz/identitymixer) on Privacy Day (January 28)

A photograph of a beach at sunset or sunrise. The ocean waves are visible in the upper left, with a white foam line. The sand is dark and wet, reflecting the ambient light. In the foreground, there are several footprints in the sand, with the most prominent one being a large, dark, well-defined print in the lower center. The text "Further Concepts" is overlaid in white on the left side of the image.

# Further Concepts



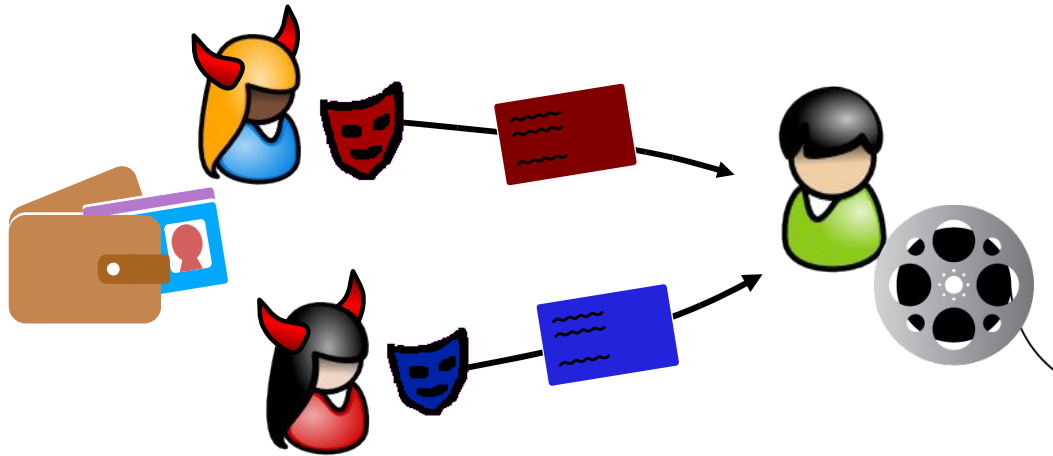


- If car is damaged: ID with insurance or gov't needs be retrieved
- Similarly: verifiably encrypt any certified attribute (*optional*)
- TTP is off-line & can be distributed to lessen trust

## Revocation authority parameters (public key)



- If Alice was speeding, license needs to be revoked!
- There are many different use cases and many solutions
  - Variants of CRL work (using crypto to maintain anonymity)
    - Accumulators
    - Signing entries & Proof, ....
  - Limited validity – certs need to be updated
  - ... For proving age, a revoked driver's license still works



Degree of anonymity can be limited:

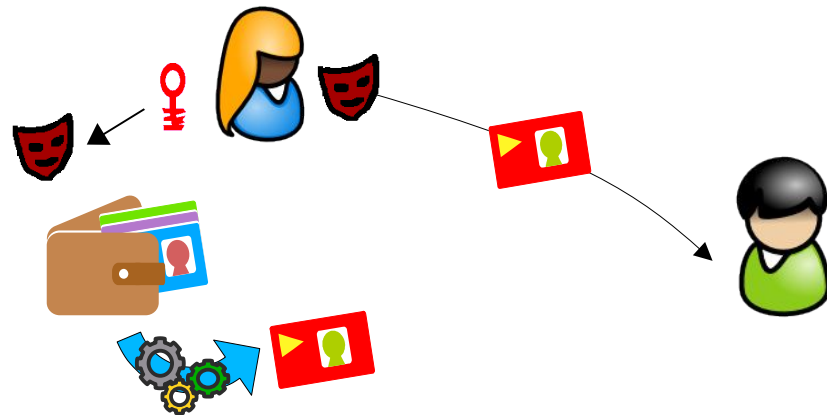
- If Alice and Eve are on-line at the same time, they are caught!
- Use Limitation – anonymous until:
  - If Alice used certs > 100 times total...
  - ... or > 10'000 times with Bob
- Alice's cert can be bound to hardware token (e.g., TPM)

A stack of approximately ten books is positioned on a dark, textured surface. The books are of varying thicknesses and colors, with some showing signs of wear. The text "A couple of use cases" is overlaid in a blue, serif font on the left side of the image.

A couple of use cases

Proving 12+, 18+, 21+ without disclosing the exact date of birth –  
privacy and compliance with age-related legislation

- Movie streaming services
- Gaming industry
- Online gambling platforms
- Dating websites
- Social benefits for young/old people





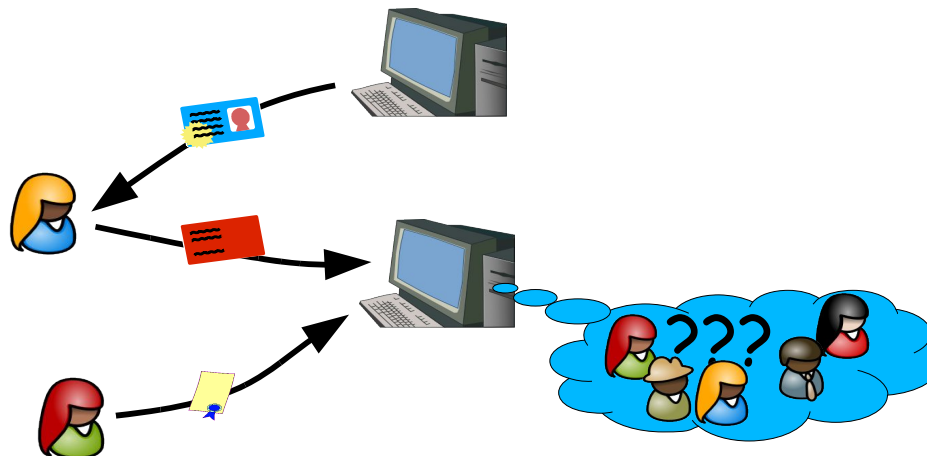
Anonymous treatment of patients (while enabling access control and payments)

- Anonymous access to patients' records
  - accessing medical test results
- Anonymous consultations with specialists
  - online chat with a psychologist
  - online consultation with IBM Watson
- Eligibility for the premium health insurance
  - proving that the body mass index (BMI) is in the certain range without disclosing the exact weight, height, or BMI



Who accesses *which data* at which time can reveal sensitive information about the users (their research strategy, location, habits, etc.)

- Patent databases
- DNA databases
- News/Journals/Magazines
- Transportation: tickets, toll roads
- Loyalty programs



Providing anonymous, but at the same time legitimate feedback

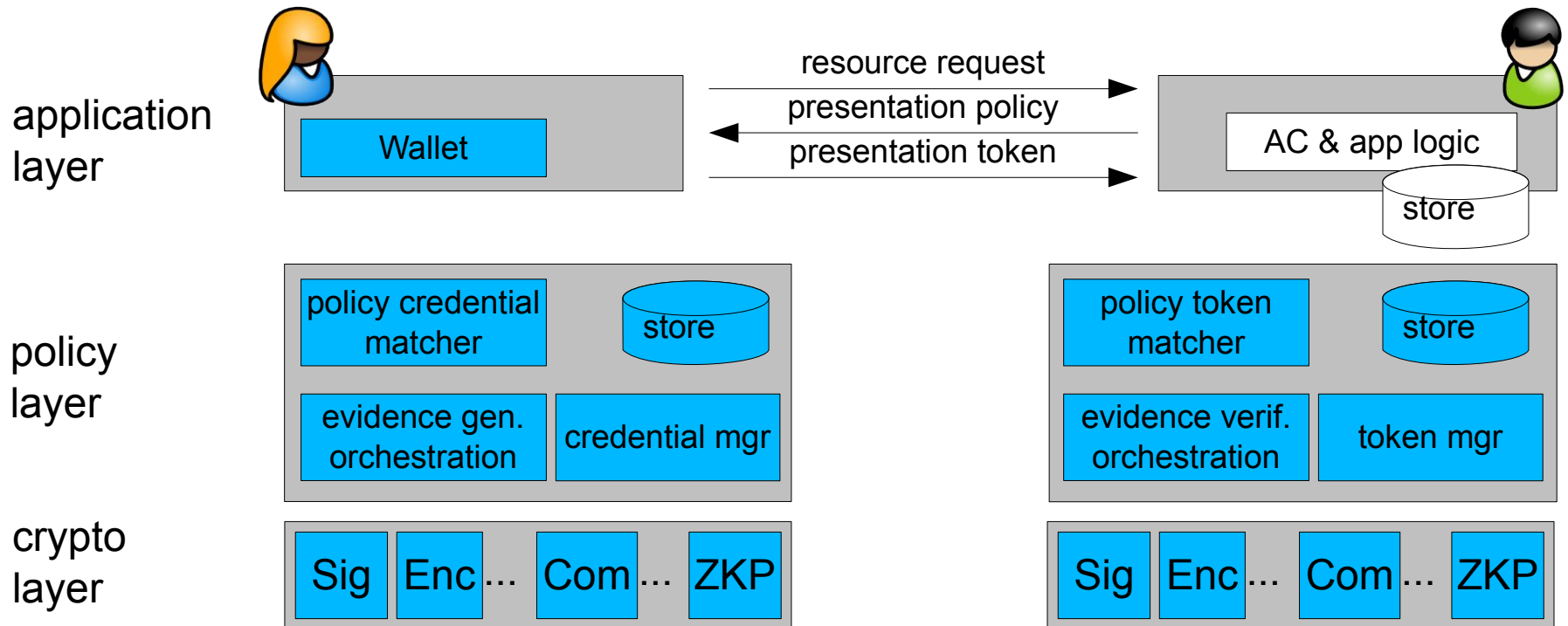
- Online polls
  - applying different restrictions on the poll participants: location, citizenship
- Rating and feedback platforms
  - anonymous feedback for a course only from the students who attended it
  - wikis
  - recommendation platforms

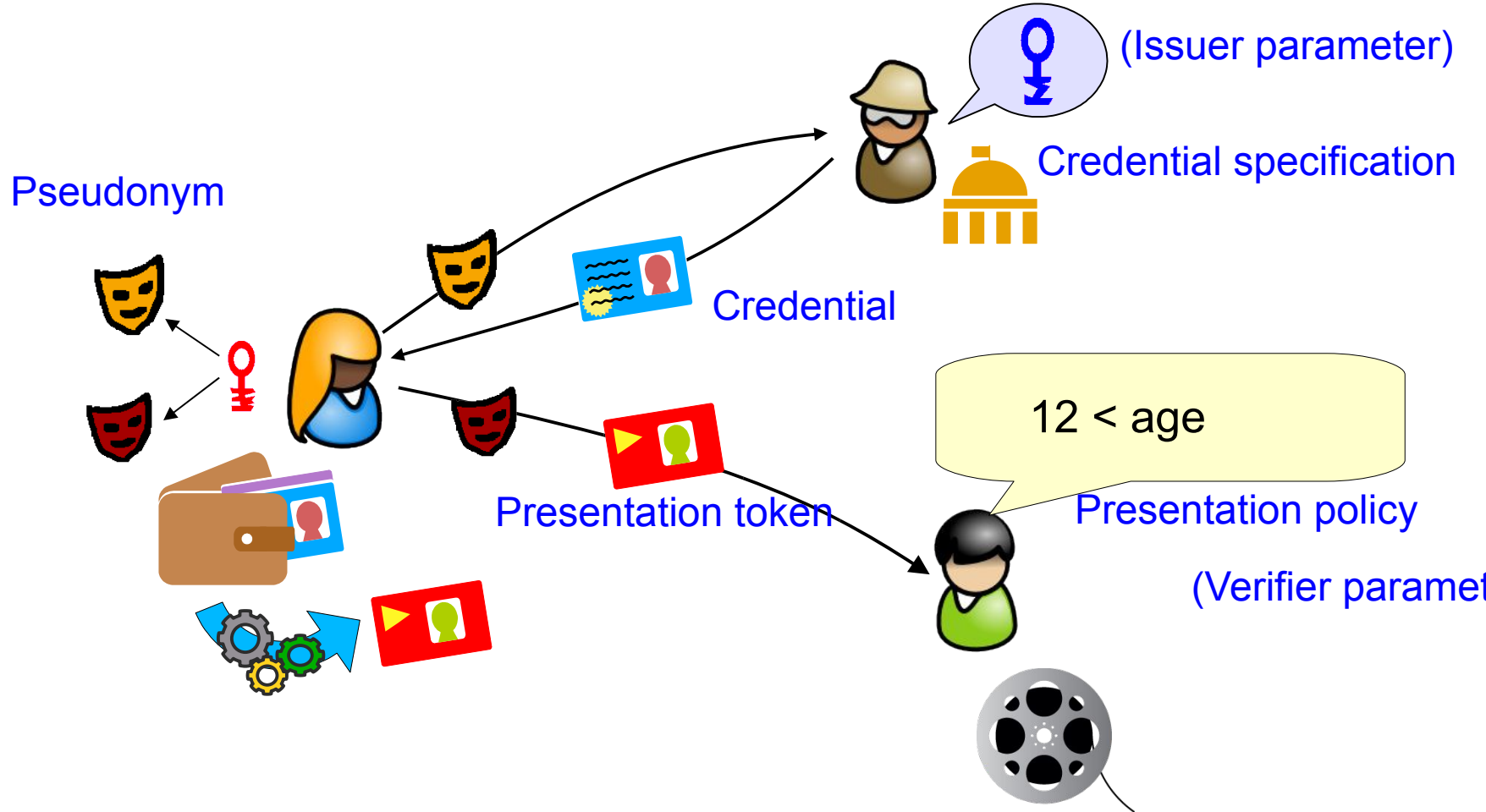


A photograph of a beach at dusk or dawn. In the foreground, a single, dark footprint is visible in the wet sand. The background shows the ocean with waves breaking, creating a white foam. The sky is a mix of orange, yellow, and blue, suggesting the time is either early morning or late evening. The overall mood is contemplative and serene.

Towards Realizing Anonymous Creds

# An Software Stack View on Identity Mixer







```
<abc:PresentationPolicy PolicyUID="https://movies...com/presentationpolicies/movie1">
```

```
<abc:Message>
```

```
<abc:ApplicationData> Terms and Conditions </abc:ApplicationData>
```

```
</abc:Message>
```

```
<abc:Credential Alias="#voucher">
```

```
<abc:CredentialSpecAlternatives>
```

```
<abc:CredentialSpecUID>https://movies....com/specifications/voucher</abc:CredentialSpecUID>
```

```
</abc:CredentialSpecAlternatives>
```

```
<abc:IssuerAlternatives>
```

```
<abc:IssuerParametersUID>https://movies....com/parameters/voucher</abc:IssuerParametersUID>
```

```
</abc:IssuerAlternatives>
```

```
</abc:Credential>
```

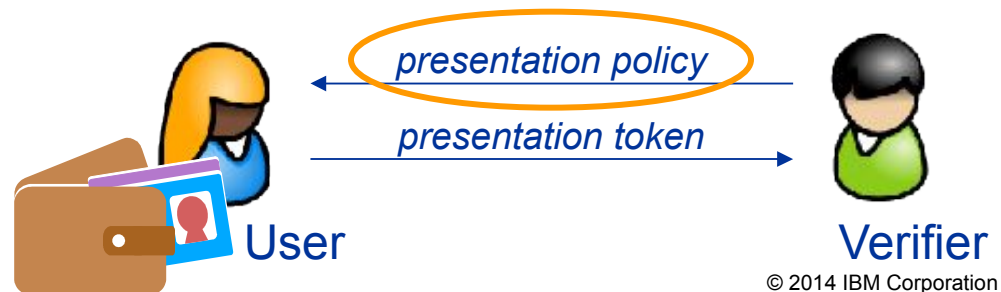
```
<abc:AttributePredicate Function="urn:oasis:names:tc:xacml:1.0:function:dateTime-geq">
```

```
<abc:Attribute CredentialAlias="#voucher" AttributeType="Expires" />
```

```
<abc:ConstantValue>2014-06-17T14:06:00Z</abc:ConstantValue>
```

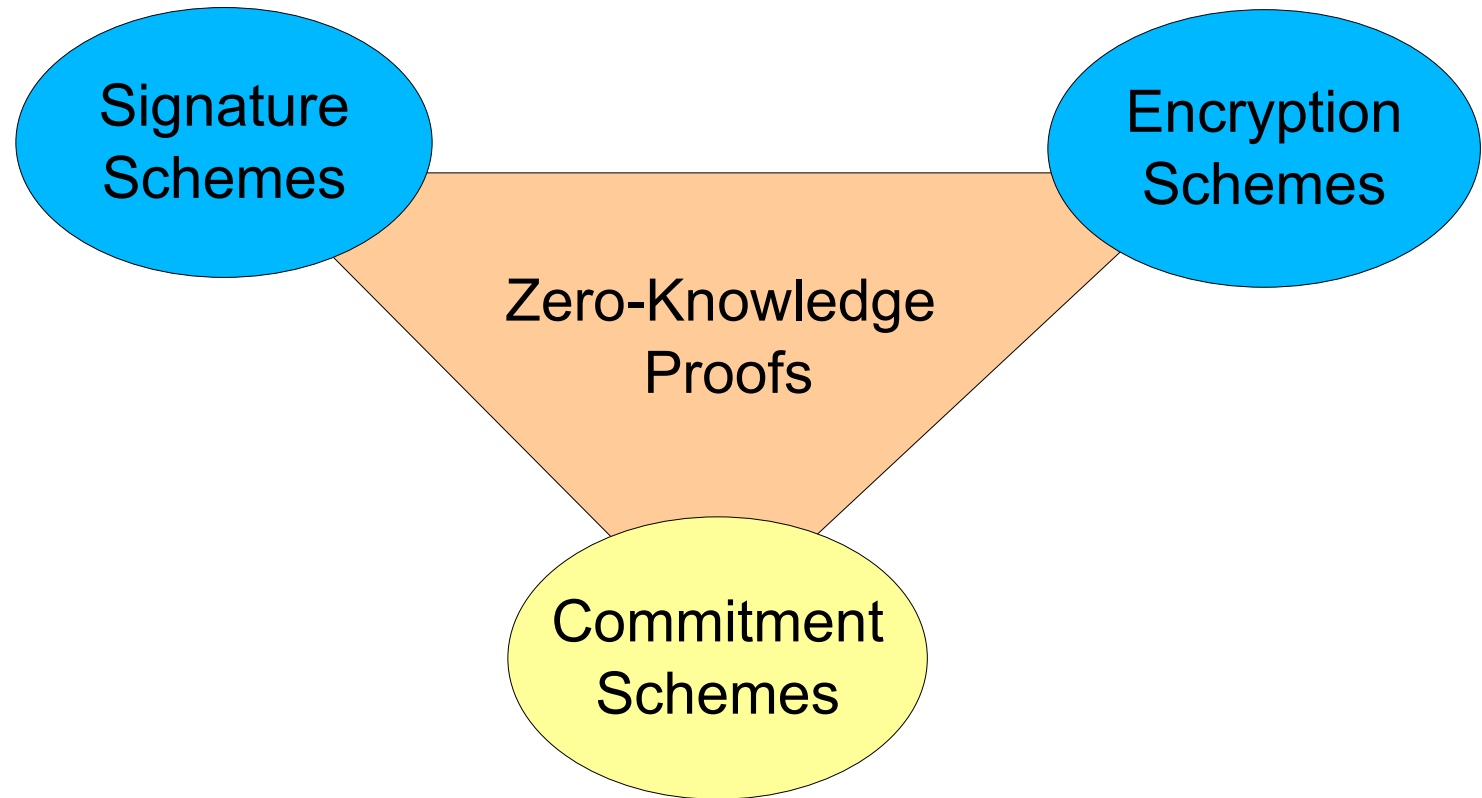
```
</abc:AttributePredicate>
```

```
</abc:PresentationPolicy>
```



A photograph of a beach at sunset or sunrise. The ocean waves are visible in the upper half of the frame, with a white foam line. The foreground is a wide expanse of dark, wet sand. In the lower center of the frame, there is a single, dark, well-defined footprint. The text "So let's look at the cryptography" is overlaid in white, sans-serif font, centered horizontally and positioned in the middle of the image.

So let's look at the cryptography

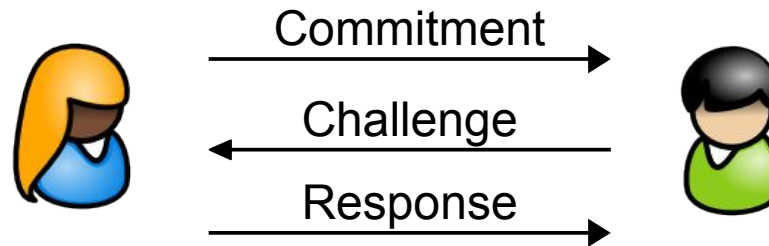


..... challenge is to do all this efficiently!

A stack of several books is placed on a dark, textured surface. The books are of various thicknesses and colors, with some showing signs of wear. The text "zero-knowledge proofs" is overlaid in a blue, serif font on the left side of the image.

zero-knowledge proofs

- interactive proof between a prover and a verifier about the prover's knowledge



- properties:

## zero-knowledge

verifier learns nothing about the prover's secret

## proof of knowledge (soundness)

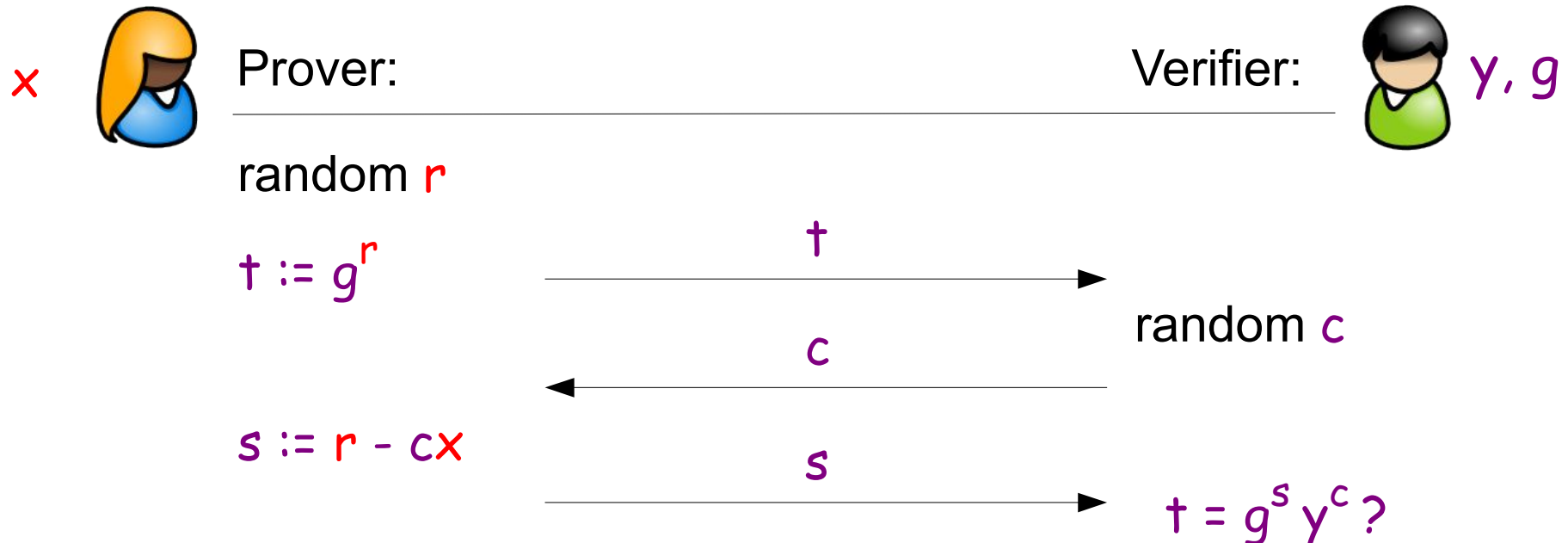
prover can convince verifier only if she knows the secret

## completeness

if prover knows the secret she can always convince the verifier

Given group  $\langle g \rangle$  and element  $y \in \langle g \rangle$ .

Prover wants to convince verifier that she *knows*  $x$  s.t.  $y = g^x$  such that verifier only learns  $y$  and  $g$ .



notation:  $PK\{(a): y = g^a\}$

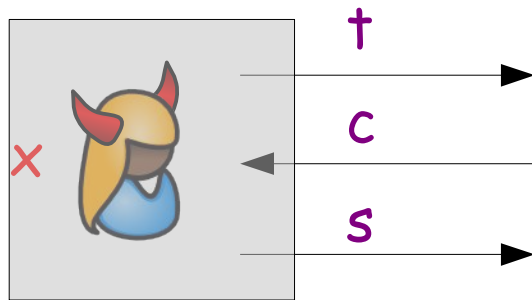


Proof of knowledge: if a prover can successfully convince a verifier, then the secret need to be extractable.

Prover might do protocol computation in any way it wants & we cannot analyse code.

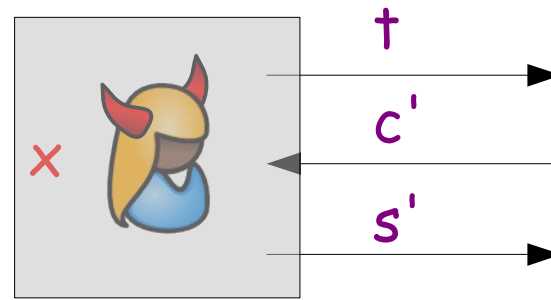
Thought experiment:

- Assume we have prover as a black box → we can reset and rerun prover
- Need to show how secret can be extracted via protocol interface



$$t = g^s y^c = g^{s'} y^{c'}$$

→



$$y^{c'-c} = g^{s-s'}$$

→

$$y = g^{(s-s')/(c'-c)}$$

→

$$x = (s-s')/(c'-c) \bmod q$$

Zero-knowledge property:

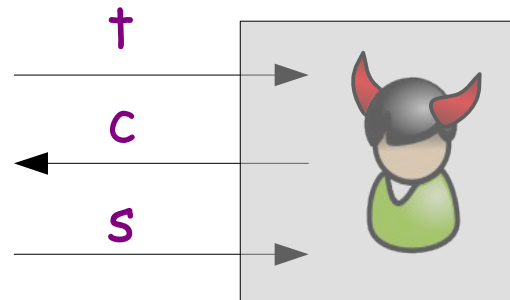
If verifier does not learn anything (except the fact that Alice knows  $x = \log_g y$ )

Idea: One can simulate whatever Bob “sees”.

Choose random  $c', s'$

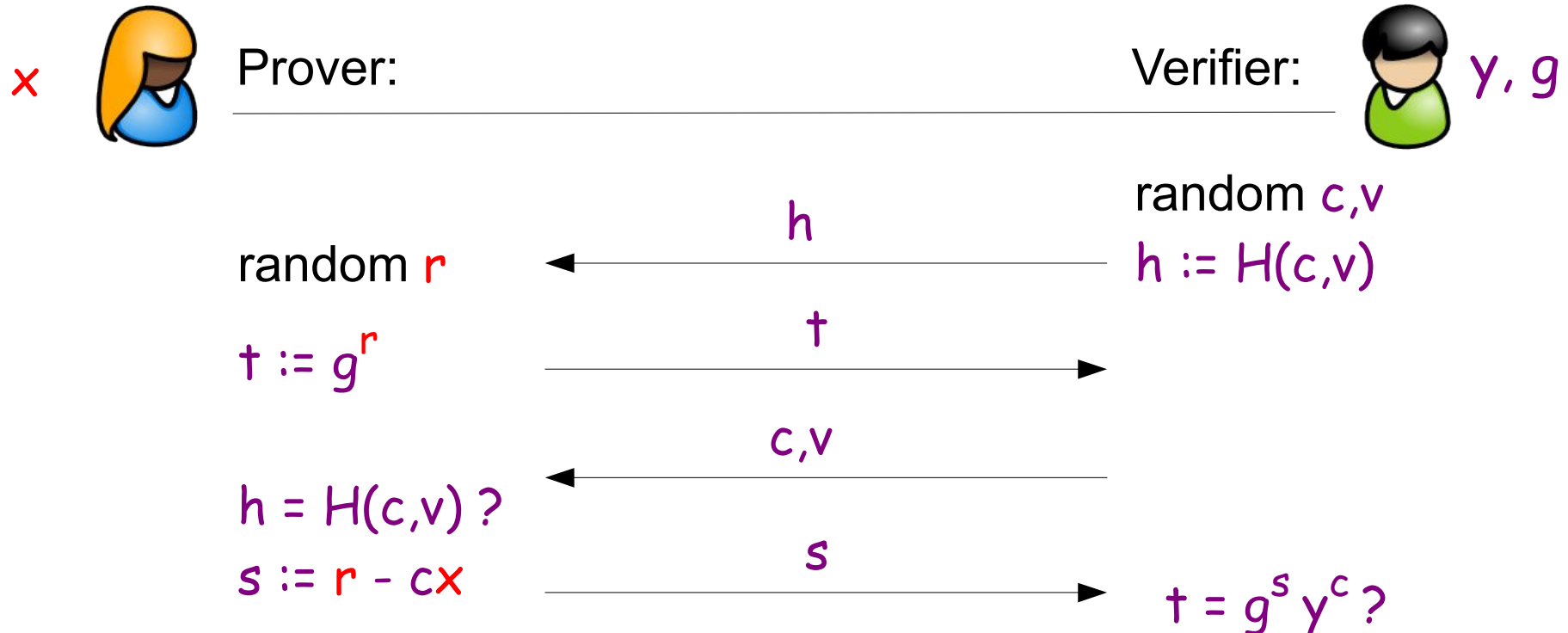
compute  $t := g^{s'} y^{c'}$

if  $c = c'$  send  $s' = s$ ,  
otherwise restart



Problem: if domain of  $c$  too large, success probability becomes too small

One way to modify protocol to get large domain  $c$ :



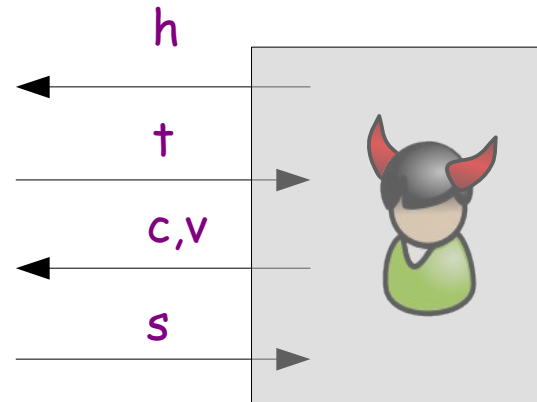
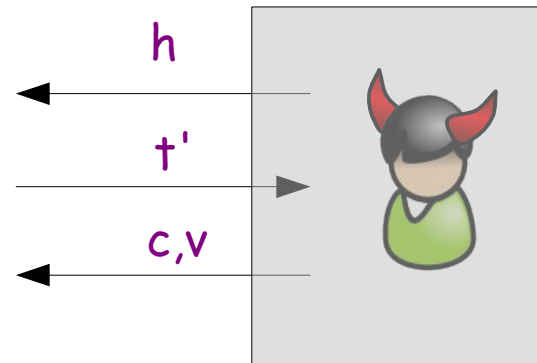
notation:  $PK\{(a): y = g^a\}$

One way to modify protocol to get large domain  $c$ :

Choose random  $c', s'$   
compute  $t' := g^{s'} y^{c'}$

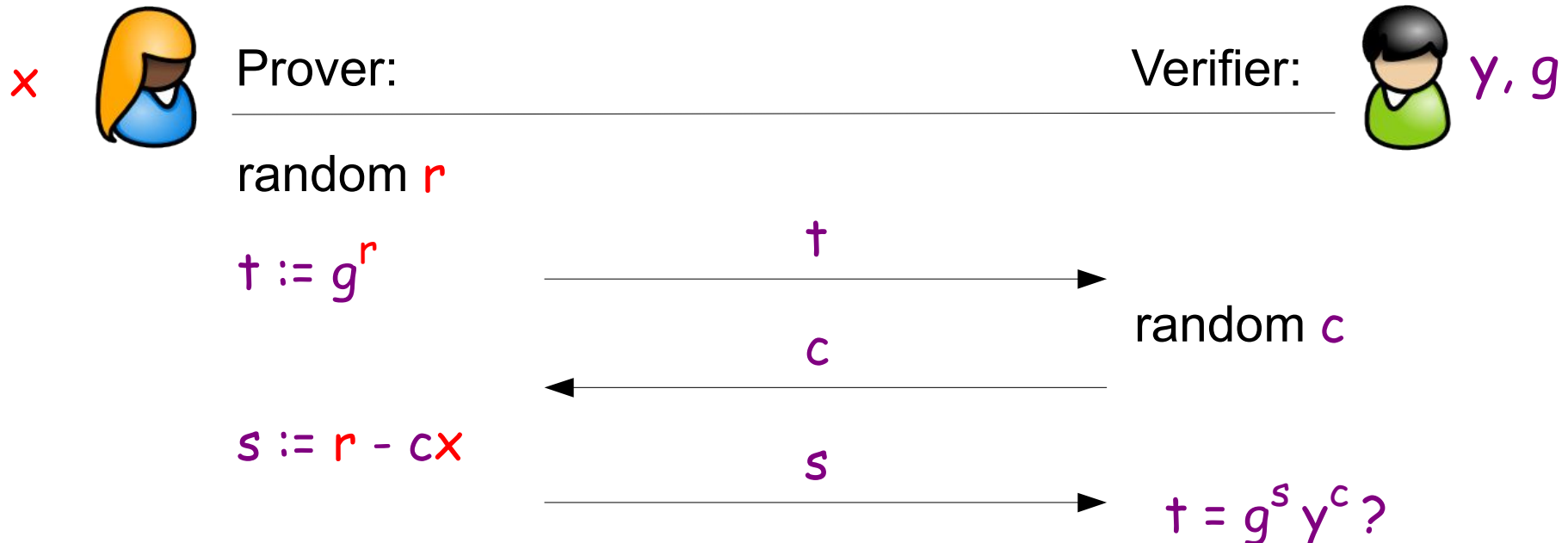
after having received  $c$   
"reboot" verifier

Choose random  $s$   
compute  $t := g^s y^c$   
send  $s$



Given group  $\langle g \rangle$  and element  $y \in \langle g \rangle$ .

Prover wants to convince verifier that she *knows*  $x$  s.t.  $y = g^x$  such that verifier only learns  $y$  and  $g$ .



notation:  $PK\{(a): y = g^a\}$

Signature SPK{(a):  $y = g^a$ }(m):

Signing a message  $m$ :

- chose random  $r \in \mathbb{Z}_q$  and
- compute  $c := H(g^r || m) = H(t || m)$   
 $s := r - cx \text{ mod } (q)$
- output  $(c, s)$



Verifying a signature  $(c, s)$  on a message  $m$ :

- check  $c = H(g^s y^c || m) ? \leftrightarrow t = g^s y^c ?$



Security:

- underlying protocol is zero-knowledge proof of knowledge
- hash function  $H(.)$  behaves as a “random oracle.”



Many Exponents:

$$\text{PK}\{(\alpha, \beta, \gamma, \delta): \gamma = g^\alpha h^\beta z^\gamma k^\delta u^\beta\}$$

Logical combinations:

$$\text{PK}\{(\alpha, \beta): \gamma = g^\alpha \wedge z = g^\beta \wedge u = g^\beta h^\alpha\}$$

$$\text{PK}\{(\alpha, \beta): \gamma = g^\alpha \vee z = g^\beta\}$$

Intervals and groups of different order (under SRSA):

$$\text{PK}\{(\alpha): \gamma = g^\alpha \wedge \alpha \in [A, B]\}$$

$$\text{PK}\{(\alpha): \gamma = g^\alpha \wedge z = g^\alpha \wedge \alpha \in [0, \min\{\text{ord}(g), \text{ord}(g)\}]\}$$

Non-interactive (Fiat-Shamir heuristic, Schnorr Signatures):

$$\text{SPK}\{(\alpha): \gamma = g^\alpha\}(m)$$

Let  $g, h, C1, C2, C3$  be group elements.

Now, what does

$$\text{PK}\{(\alpha_1, \beta_1, \alpha_2, \beta_2, \alpha_3, \beta_3): C1 = g^{\alpha_1} h^{\beta_1} \wedge C2 = g^{\alpha_2} h^{\beta_2} \wedge C3 = g^{\alpha_3} h^{\beta_3} \wedge C3 = g^{\alpha_1} g^{\alpha_2} h^{\beta_3}\}$$

mean?

→ Prover knows values  $\alpha_1, \beta_1, \alpha_2, \beta_2, \beta_3$  such that

$$C1 = g^{\alpha_1} h^{\beta_1}, \quad C2 = g^{\alpha_2} h^{\beta_2} \quad \text{and}$$

$$C3 = g^{\alpha_1} g^{\alpha_2} h^{\beta_3} = g^{\alpha_1 + \alpha_2} h^{\beta_3} = g^{\alpha_3} h^{\beta_3}$$

$$\rightarrow \alpha_3 = \alpha_1 + \alpha_2 \pmod{q}$$

And what about:

$$\text{PK}\{(\alpha_1, \dots, \beta_3): C1 = g^{\alpha_1} h^{\beta_1} \wedge C2 = g^{\alpha_2} h^{\beta_2} \wedge C3 = g^{\alpha_3} h^{\beta_3} \wedge C3 = g^{\alpha_1} (g^5)^{\alpha_2} h^{\beta_3}\}$$

$$\rightarrow C3 = g^{\alpha_1} g^{5\alpha_2} h^{\beta_3} = g^{\alpha_1 + 5\alpha_2} h^{\beta_3}$$

$$\rightarrow \alpha_3 = \alpha_1 + 5\alpha_2 \pmod{q}$$

Let  $g, h, C1, C2, C3$  be group elements.

Now, what does

$\text{PK}\{(\alpha_1, \dots, \beta_3): C1 = g^{\alpha_1} h^{\beta_1} \wedge C2 = g^{\alpha_2} h^{\beta_2} \wedge C3 = g^{\alpha_3} h^{\beta_3} \wedge C3 = C2^{\alpha_1} h^{\beta_3}\}$  mean?

→ Prover knows values  $\alpha_1, \beta_1, \alpha_2, \beta_2, \beta_3$  such that

$$C1 = g^{\alpha_1} h^{\beta_1}, \quad C2 = g^{\alpha_2} h^{\beta_2} \quad \text{and}$$

$$C3 = C2^{\alpha_1} h^{\beta_3} = (g^{\alpha_2} h^{\beta_2})^{\alpha_1} h^{\beta_3} = g^{\alpha_2 \cdot \alpha_1} h^{\beta_3 + \beta_2 \cdot \alpha_1}$$

$$C3 = g^{\alpha_2 \cdot \alpha_1} h^{\beta_3 + \beta_2 \cdot \alpha_1} = g^{\alpha_3} h^{\beta_3'}$$

$$\rightarrow \alpha_3 = \alpha_1 \cdot \alpha_2 \pmod{q}$$

And what about

$\text{PK}\{(\alpha_1, \beta_1, \beta_2): C1 = g^{\alpha_1} h^{\beta_1} \wedge C2 = g^{\alpha_2} h^{\beta_2} \wedge C2 = C1^{\alpha_1} h^{\beta_2}\}$

$$\rightarrow \alpha_2 = \alpha_1^2 \pmod{q}$$

Let  $g, h, C1, C2, C3$  be group elements.

Now, what does

$\text{PK}\{(\alpha_1, \dots, \beta_2): C1 = g^{\alpha_1} h^{\beta_1} \wedge C2 = g^{\alpha_2} h^{\beta_2} \wedge g = (C2/C1)^{\alpha_1} h^{\beta_2}\}$  mean?

→ Prover knows values  $\alpha, \beta_1, \beta_2$  such that

$$C1 = g^{\alpha_1} h^{\beta_1}$$

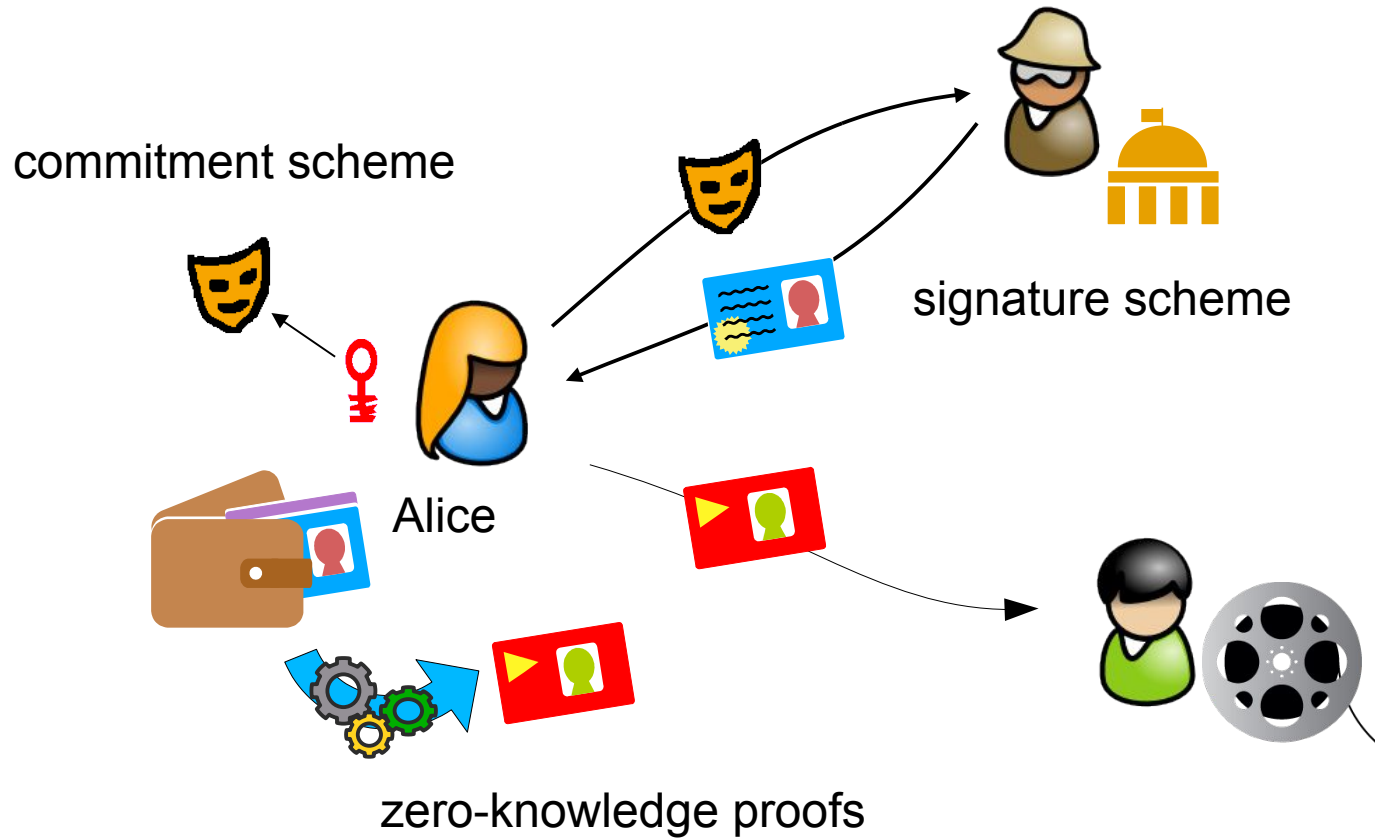
$$g = (C2/C1)^{\alpha_1} h^{\beta_2} = (C2 g^{-\alpha_1} h^{-\beta_1})^{\alpha_1} h^{\beta_2}$$

$$\rightarrow g^{1/\alpha_1} = C2 g^{-\alpha_1} h^{-\beta_1} h^{\beta_2/\alpha_1}$$

$$C2 = g^{\alpha_1} h^{\beta_1} h^{-\beta_2/\alpha_1} g^{1/\alpha_1} = g^{\alpha_1 + 1/\alpha_1} h^{\beta_1 - \beta_2/\alpha_1}$$

$$C2 = g^{\alpha_2} h^{\beta_2}$$

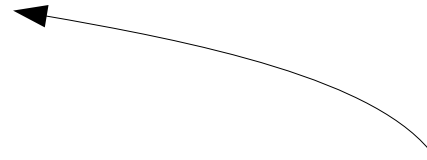
$$\alpha_2 = \alpha_1 + \alpha_1^{-1} \pmod{q}$$



A stack of several papers or documents is lying on a textured, light-colored surface. The papers are slightly fanned out, showing their edges. The background is a mottled, light brown or tan color with some darker spots.

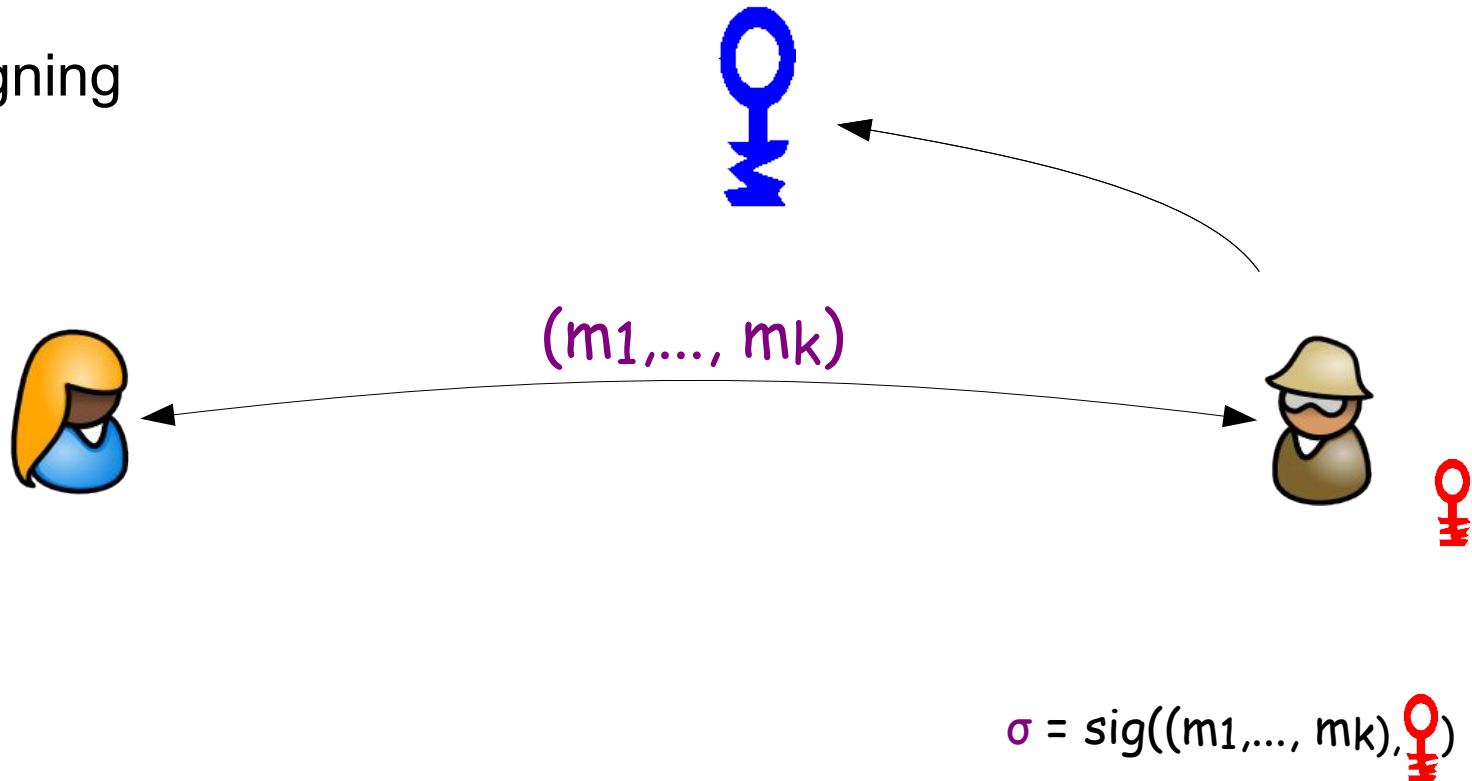
signature schemes

## Key Generation

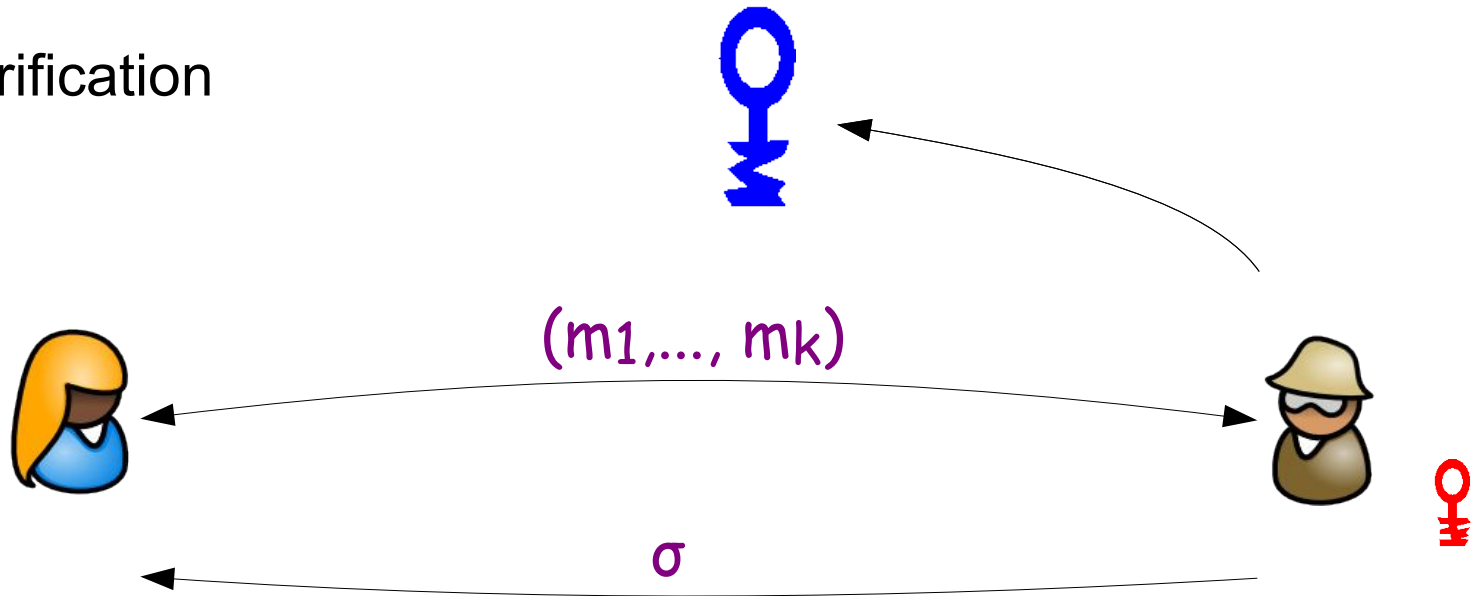




Signing



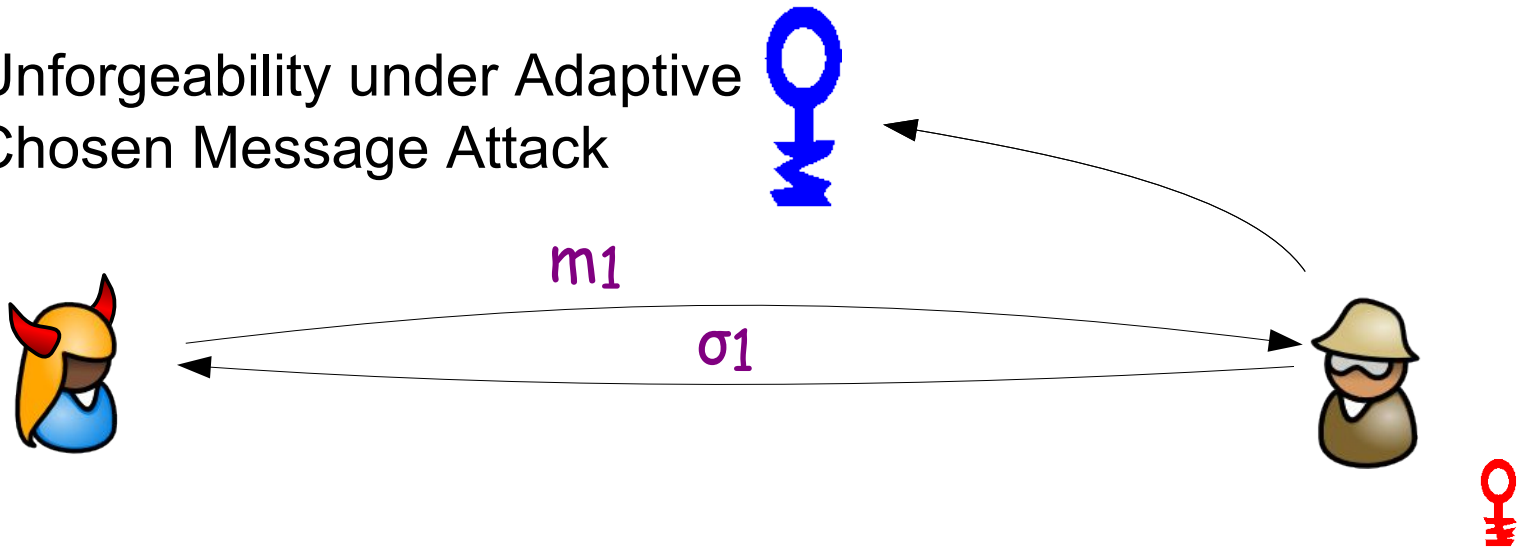
Verification



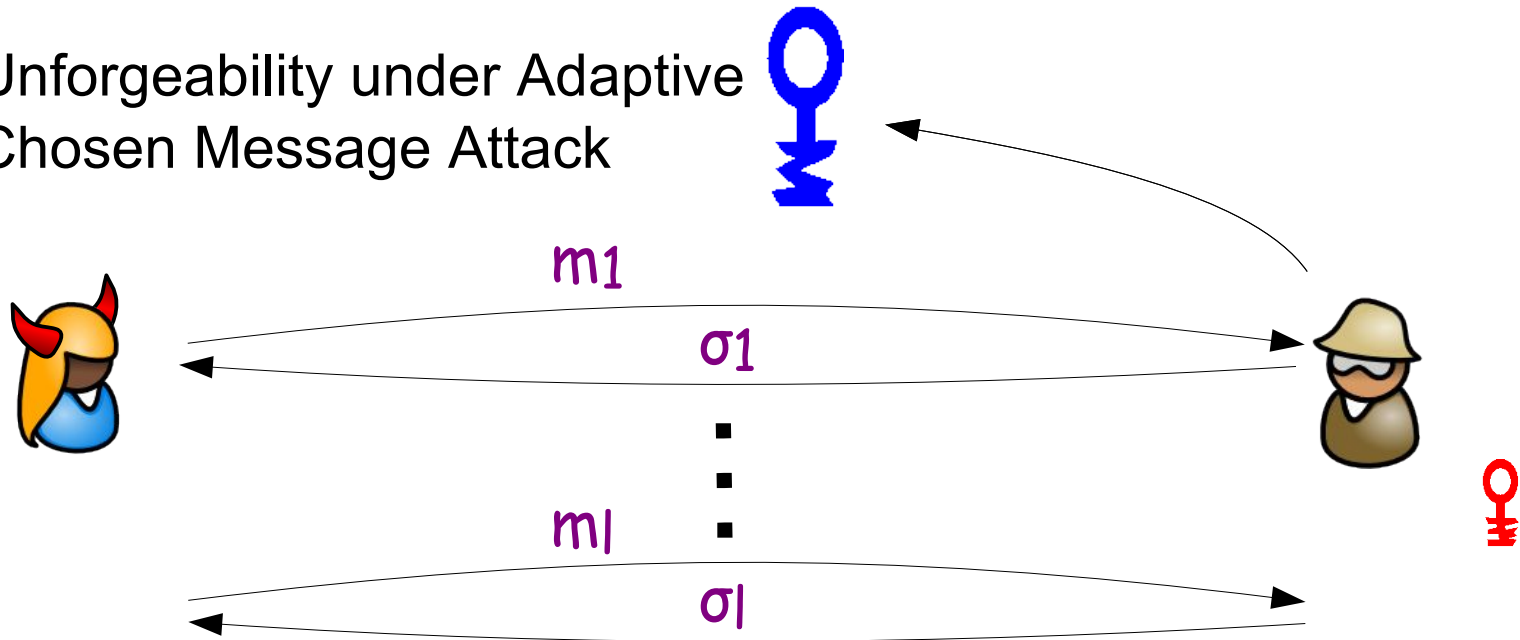
$$\sigma = \text{sig}((m_1, \dots, m_k), \text{red key})$$

$$\text{ver}(\sigma, (m_1, \dots, m_k), \text{blue key}) = \text{true}$$

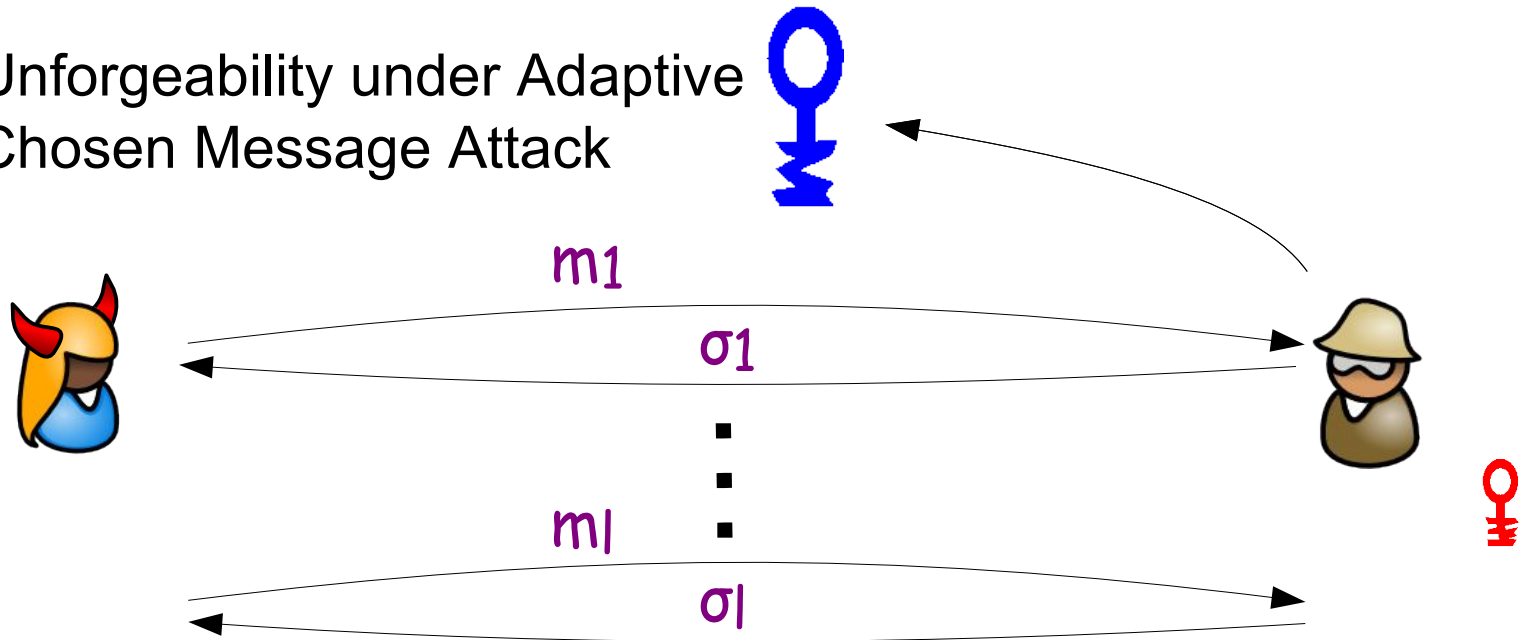
Unforgeability under Adaptive  
Chosen Message Attack



## Unforgeability under Adaptive Chosen Message Attack

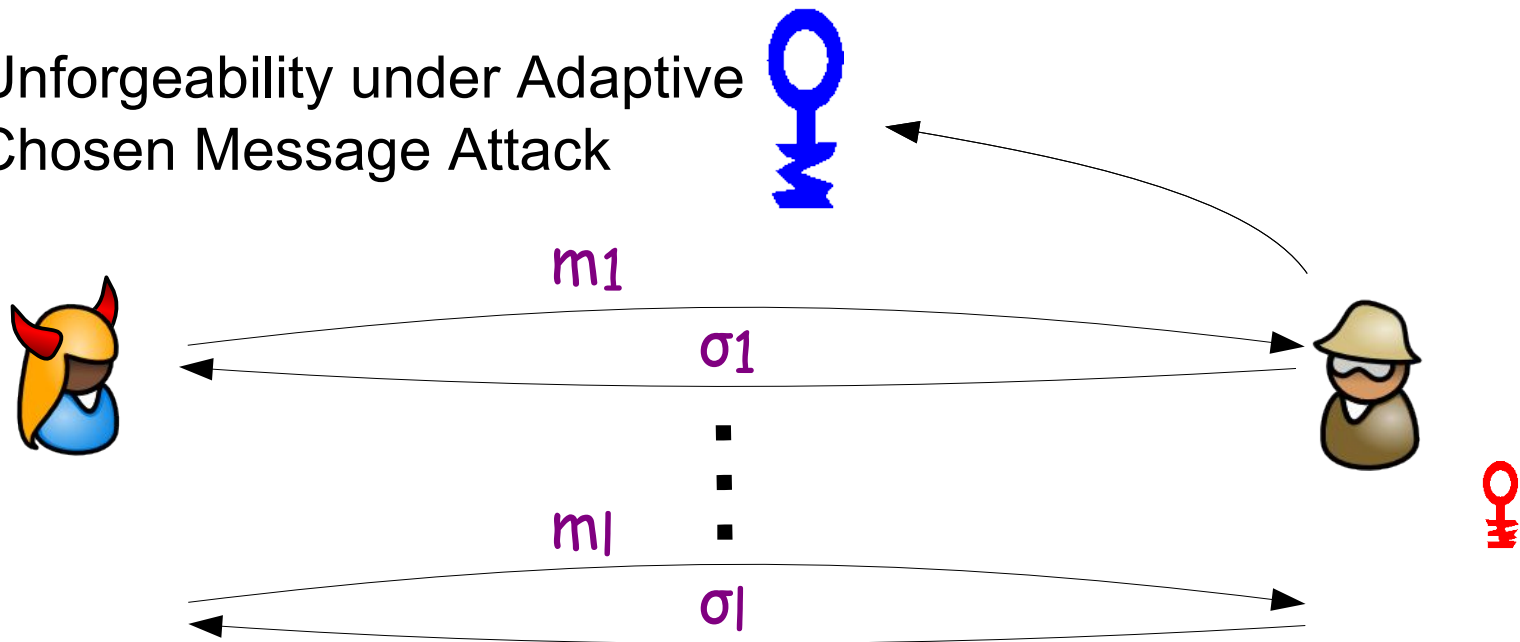


## Unforgeability under Adaptive Chosen Message Attack



$\sigma'$  and  $m' \neq m_i$  s.t.  
 $\text{ver}(\sigma', m', \text{blue key}) = \text{true}$

## Unforgeability under Adaptive Chosen Message Attack



and  $m' \neq m_i$  s.t.  
 $\text{ver}(\sigma', m', \text{blue key}) = \text{true}$

Rivest, Shamir, and Adleman 1978

Secret Key: two random primes  $p$  and  $q$

Public Key:  $n := pq$ , prime  $e$ ,  
and collision-free hash function

$$H: \{0,1\}^* \rightarrow \{0,1\}^{\ell}$$

Computing signature on a message  $m \in \{0,1\}^*$

$$d := 1/e \bmod (p-1)(q-1)$$

$$s := H(m)^d \bmod n$$



Verification of signature  $s$  on a message  $m \in \{0,1\}^*$

$$s^e = H(m) \pmod{n}$$

$$\text{Correctness: } s^e = (H(m)^d)^e = H(m)^{d \cdot e} = H(m) \pmod{n}$$

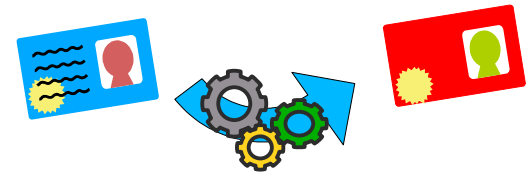
Verification signature on a message  $m \in \{0,1\}^*$

$$s^e := H(m) \pmod{n}$$



Wanna do proof of knowledge of signature on a message, e.g.,

$$\text{PK}\{ (m,s): s^e = H(m) \pmod{n} \}$$



But this is not a valid proof expression!!!! :-)



Public key of signer: RSA modulus  $n$  and  $a_i, b, d \in \mathbb{Q}\mathbb{R}_n$ , 

Secret key: factors of  $n$

To sign  $k$  messages  $m_1, \dots, m_k \in \{0,1\}^\ell$  :

- choose random *prime*  $2^{\ell+2} > e > 2^{\ell+1}$  and *integer*  $s \approx n$
- compute  $c$  :

$$c = (d / (a_1^{m_1} \cdot \dots \cdot a_k^{m_k} b^s))^{1/e} \bmod n$$

- signature is  $(c, e, s)$



To verify a signature  $(c, e, s)$  on messages  $m_1, \dots, m_k$ :

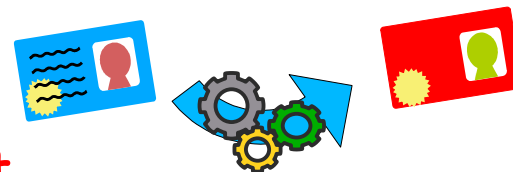
- $m_1, \dots, m_k \in \{0,1\}^\ell$ :
- $e > 2^{\ell+1}$
- $d = c^e a_1^{m_1} \dots a_k^{m_k} b^s \bmod n$



Theorem: *Signature scheme is secure against adaptively chosen message attacks under Strong RSA assumption.*

Recall:  $d = c^e a_1^{m_1} a_2^{m_2} b^s \pmod n$

Observe:

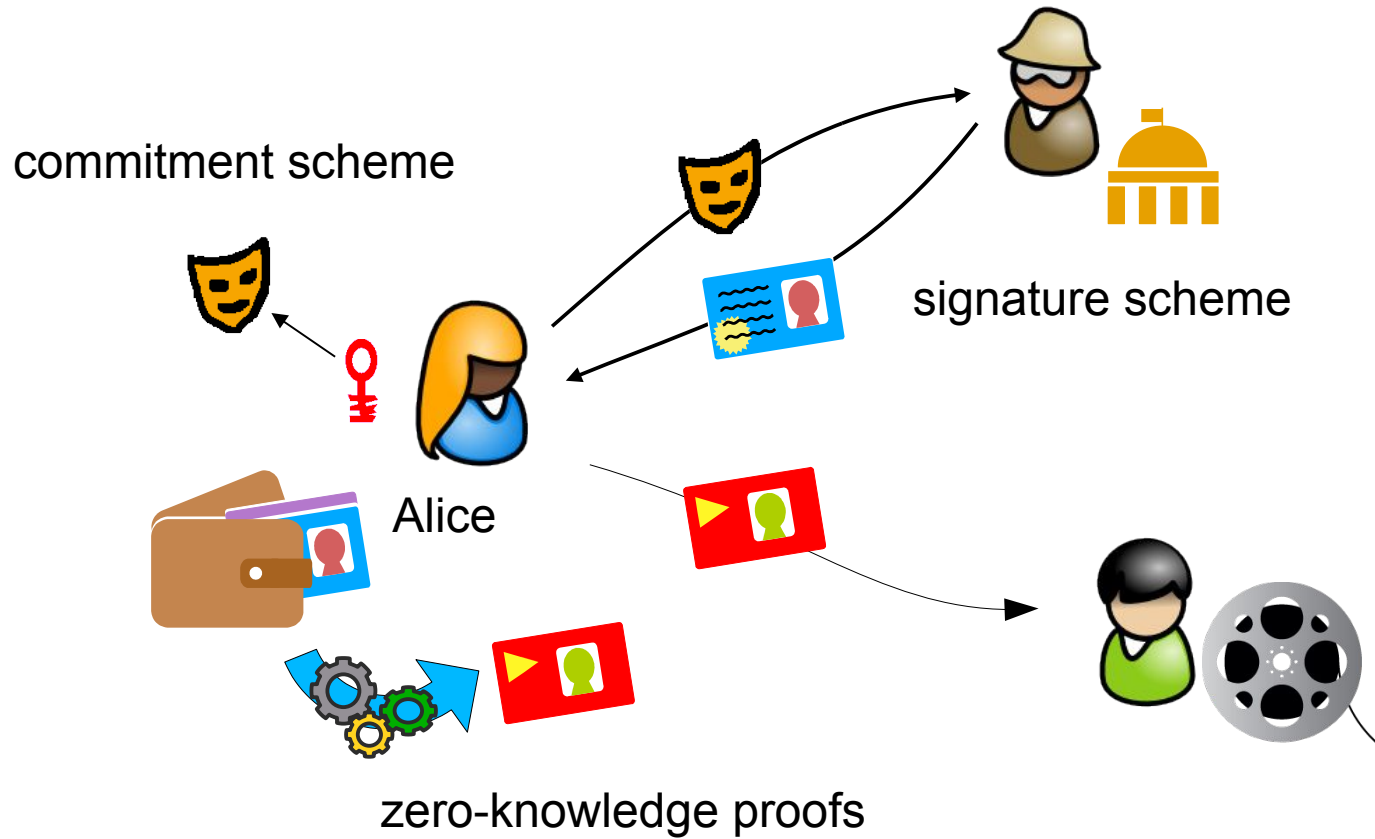


- Let  $c' = c b^t \pmod n$  with randomly chosen  $t$
- Then  $d = c'^e a_1^{m_1} a_2^{m_2} b^{s-et} \pmod n$ , i.e.,  $(c', e, s^* = s-et)$  is also signature on  $m_1$  and  $m_2$

To prove knowledge of signature  $(c', e, s^*)$  on  $m_2$  and some  $m_1$

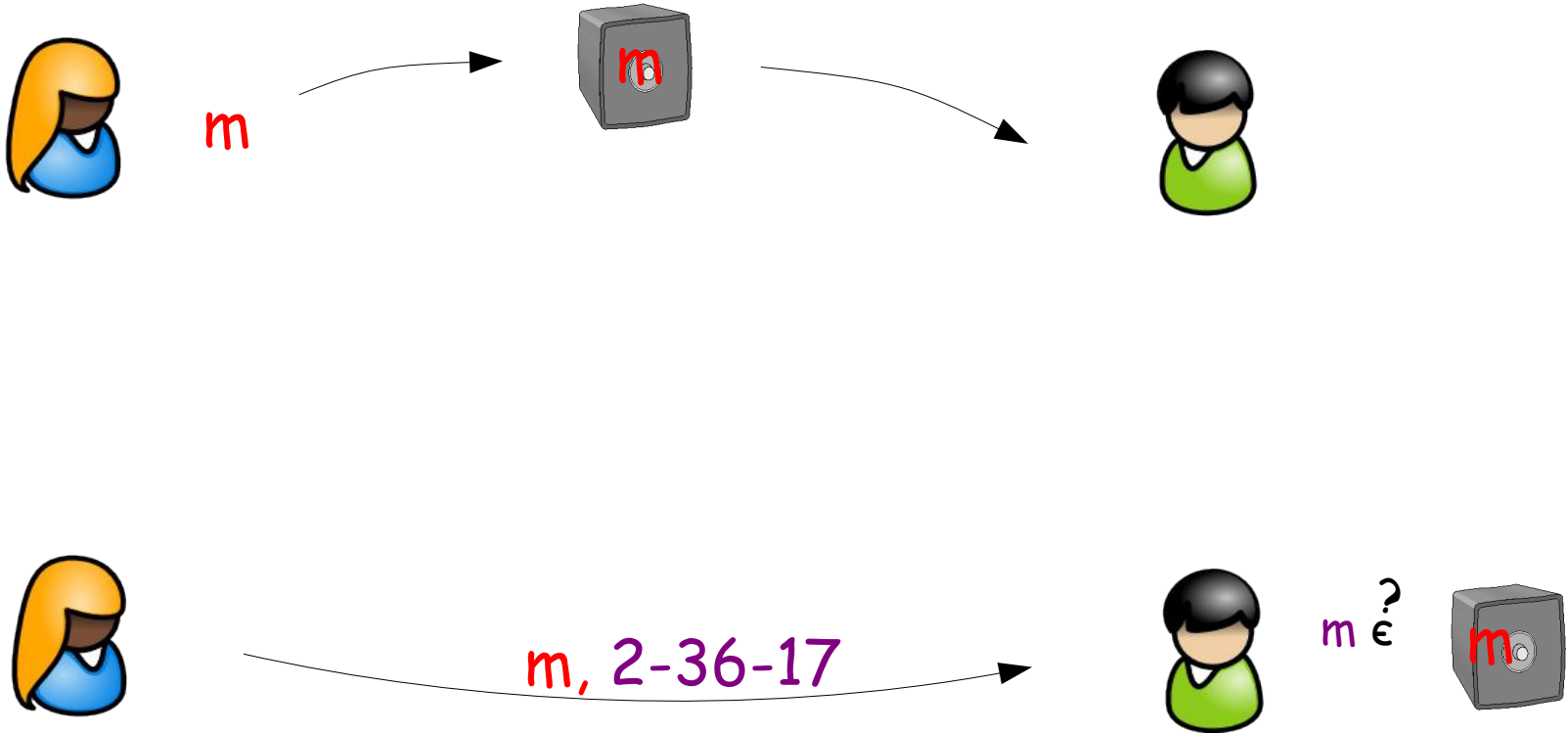
- provide  $c'$
- $\text{PK}\{(\epsilon, \mu_1, \sigma) : d/a_2^{m_2} := c'^\epsilon a_1^{\mu_1} b^\sigma \wedge \mu_1 \in \{0,1\}^\ell \wedge \epsilon > 2^{\ell+1}\}$

→ proves  $d := c'^\epsilon a_1^{\mu_1} a_2^{m_2} b^\sigma$

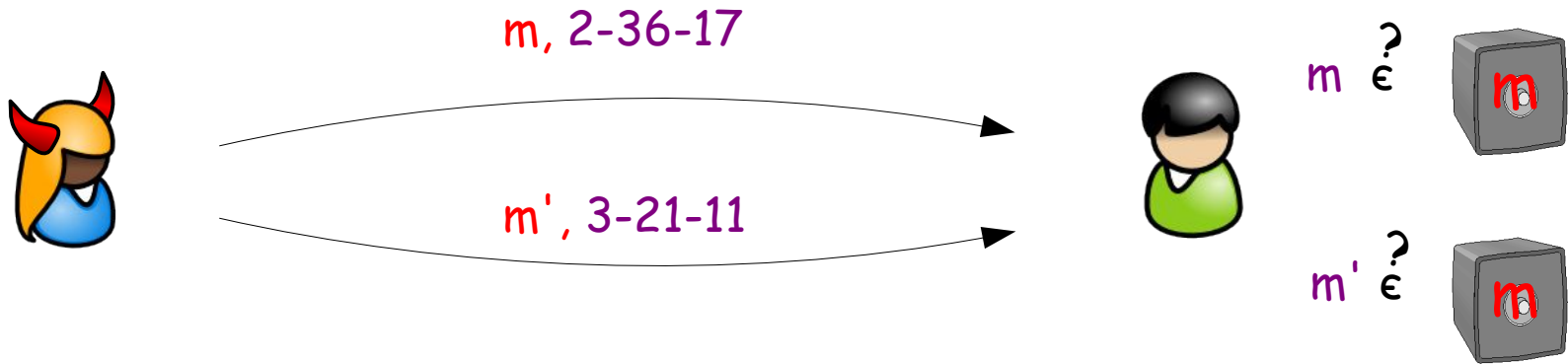


A stack of several books is positioned on a dark, textured surface. The books are of varying thicknesses and colors, with some showing signs of wear. The background is a mottled, dark grey or black surface with some lighter, brownish patches. The text "commitment scheme" is overlaid on the left side of the image in a blue, sans-serif font.

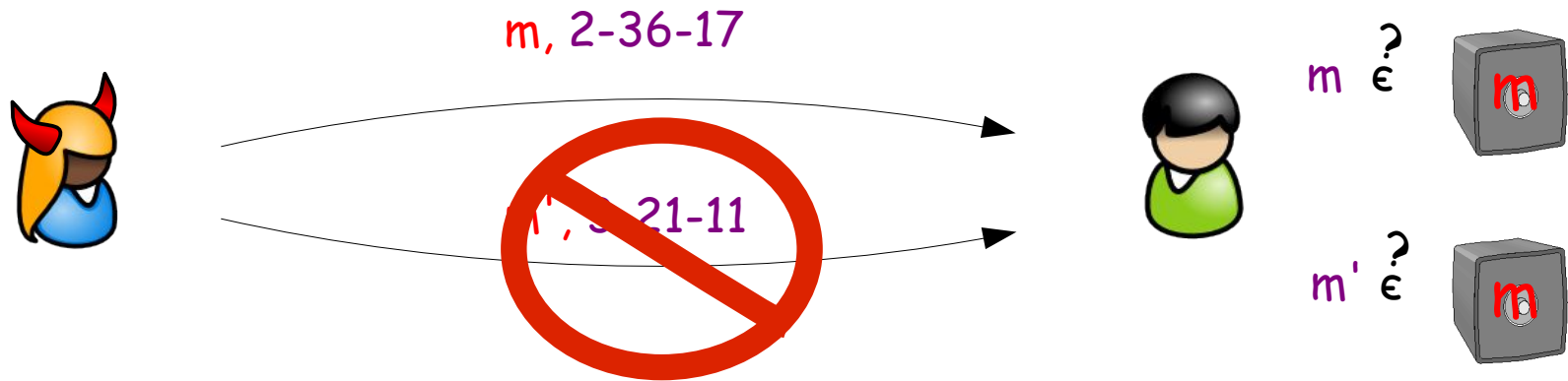
commitment scheme



## Binding

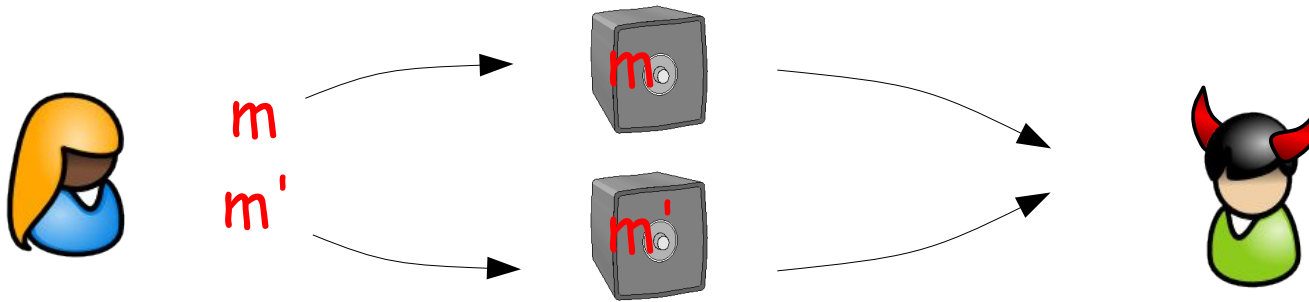


## Binding

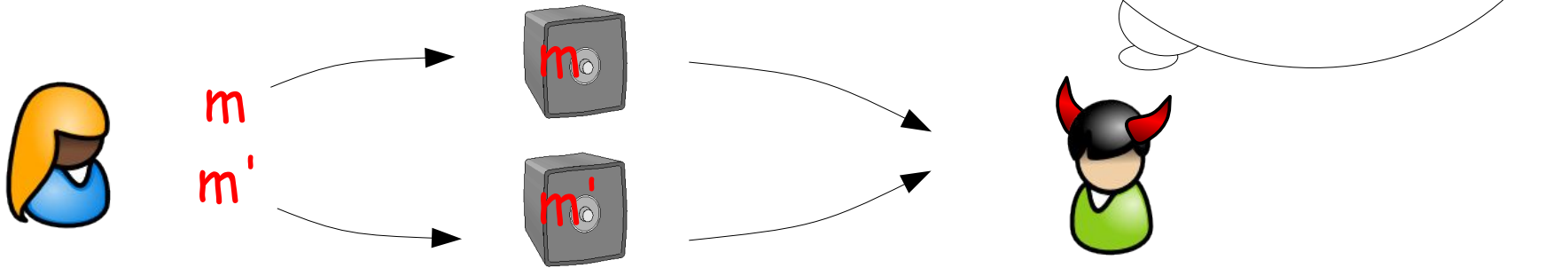




Hiding: for all message  $m, m'$



Hiding: for all message  $m, m'$



Group  $G = \langle g \rangle = \langle h \rangle$  of order  $q$

To commit to element  $x \in \mathbb{Z}_q$ :

- Pedersen: perfectly hiding, computationally binding

choose  $r \in \mathbb{Z}_q$  and compute  $c = g^x h^r$

- ElGamal: computationally hiding, perfectly binding:

choose  $r \in \mathbb{Z}_q$  and compute  $c = (g^x h^r, g^r)$

To open commitment:

- reveal  $x$  and  $r$  to verifier
- verifier checks if  $c = g^x h^r$

Pedersen's Scheme:

Choose  $r \in \mathbb{Z}_q$  and compute  $c = g^x h^r$

Perfectly hiding:

Let  $c$  be a commitment and  $u = \log_g h$

$$\begin{aligned}\text{Thus } c &= g^x h^r = g^{x+ur} = g^{(x+ur') + u(r-r')} \\ &= g^{x+ur'} h^{r-r'} \quad \text{for any } r'!\end{aligned}$$

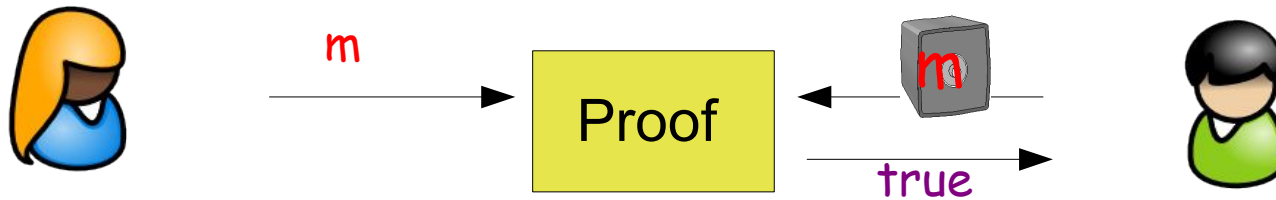
I.e., given  $c$  and  $x'$  here exist  $r'$  such that  $c = g^{x'} h^{r'}$

Computationally binding:

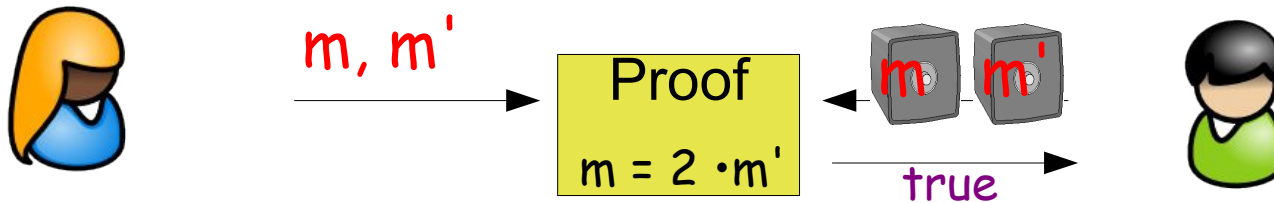
Let  $c, (x', r')$  and  $(x, r)$  s.t.  $c = g^{x'} h^{r'} = g^x h^r$

Then  $g^{x'-x} = h^{r-r'}$  and  $u = \log_g h = (x'-x)/(r-r') \bmod q$

## Proof of Knowledge of Contents



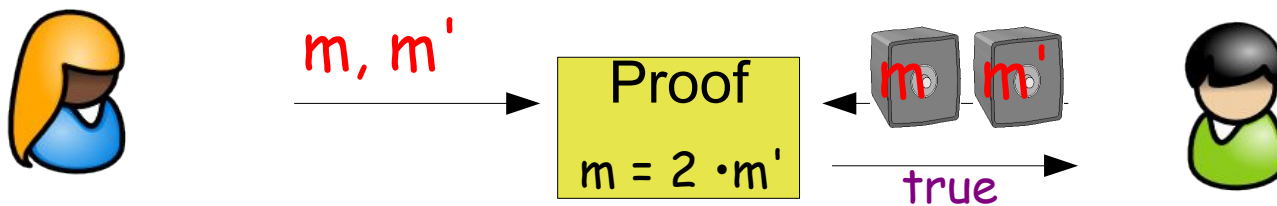
## Proof of Relations among Contents



Let  $C = g^m h^r$  and  $C' = g^{m'} h^r$  then:



$PK\{(a, \beta): C = g^\beta h^a\}$

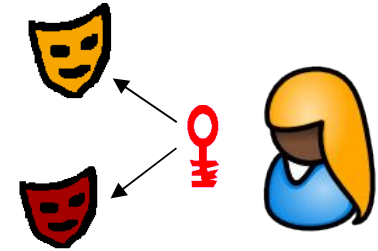


$PK\{(a, \beta, \gamma): C' = g^\beta h^a \wedge C = (g^2)^\beta h^\gamma\}$

A wooden crate, filled with books, is lying on its side on a textured, light-colored surface. The crate is made of light-colored wood and is filled with several books of various sizes and colors. The books are stacked in a way that their spines are visible. The surface the crate is on appears to be a rough, light-colored material, possibly concrete or stone. The lighting is bright, casting a shadow of the crate onto the surface to its right.

putting things together

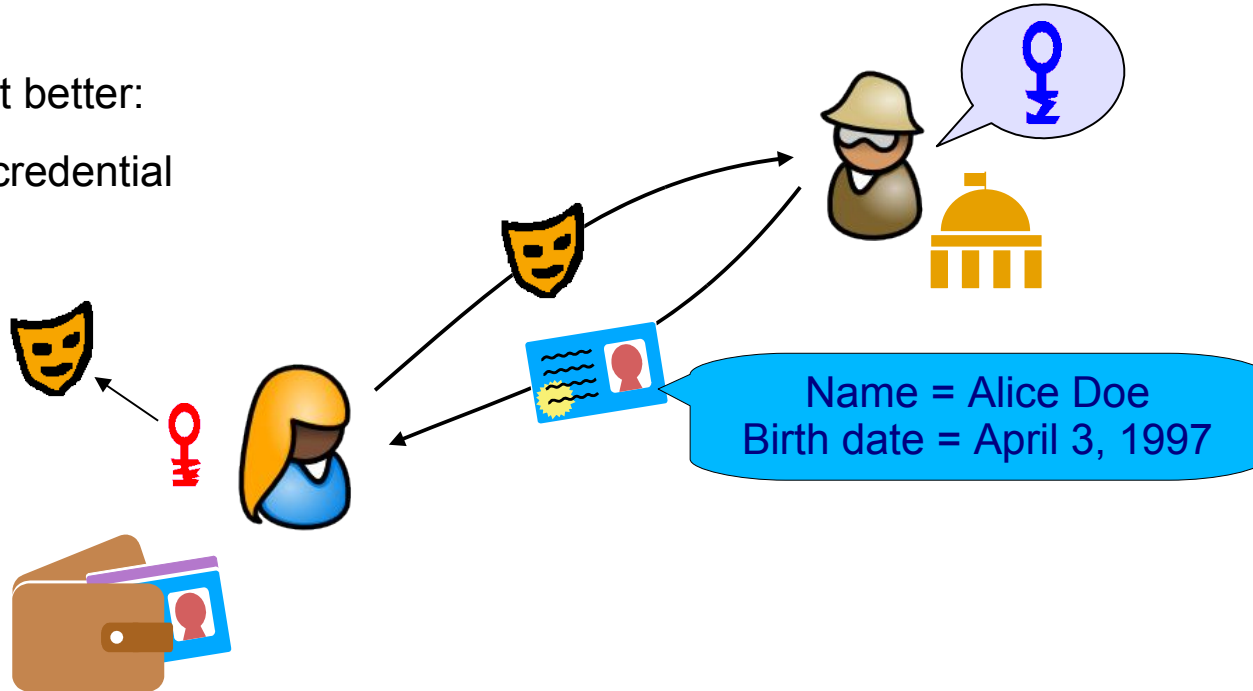
- Let  $G = \langle g \rangle = \langle h \rangle$  of order  $q$
- User's secret key: random  $sk \in \mathbb{Z}_q$
- To compute a pseudonym  $Nym$ 
  - Choose random  $r \in \mathbb{Z}_q$
  - Compute  $Nym = g^{sk} h^r$





Like PKI, but better:

- Issuing a credential



*Concept: credentials*

Recall: a signature  $(c, e, s)$  on messages  $m_1, \dots, m_k$ :

$$- m_1, \dots, m_k \in \{0,1\}^\ell:$$

$$- e > 2^{\ell+1}$$

$$- d = c^e a_1^{m_1} \dots a_k^{m_k} b^s \bmod n$$

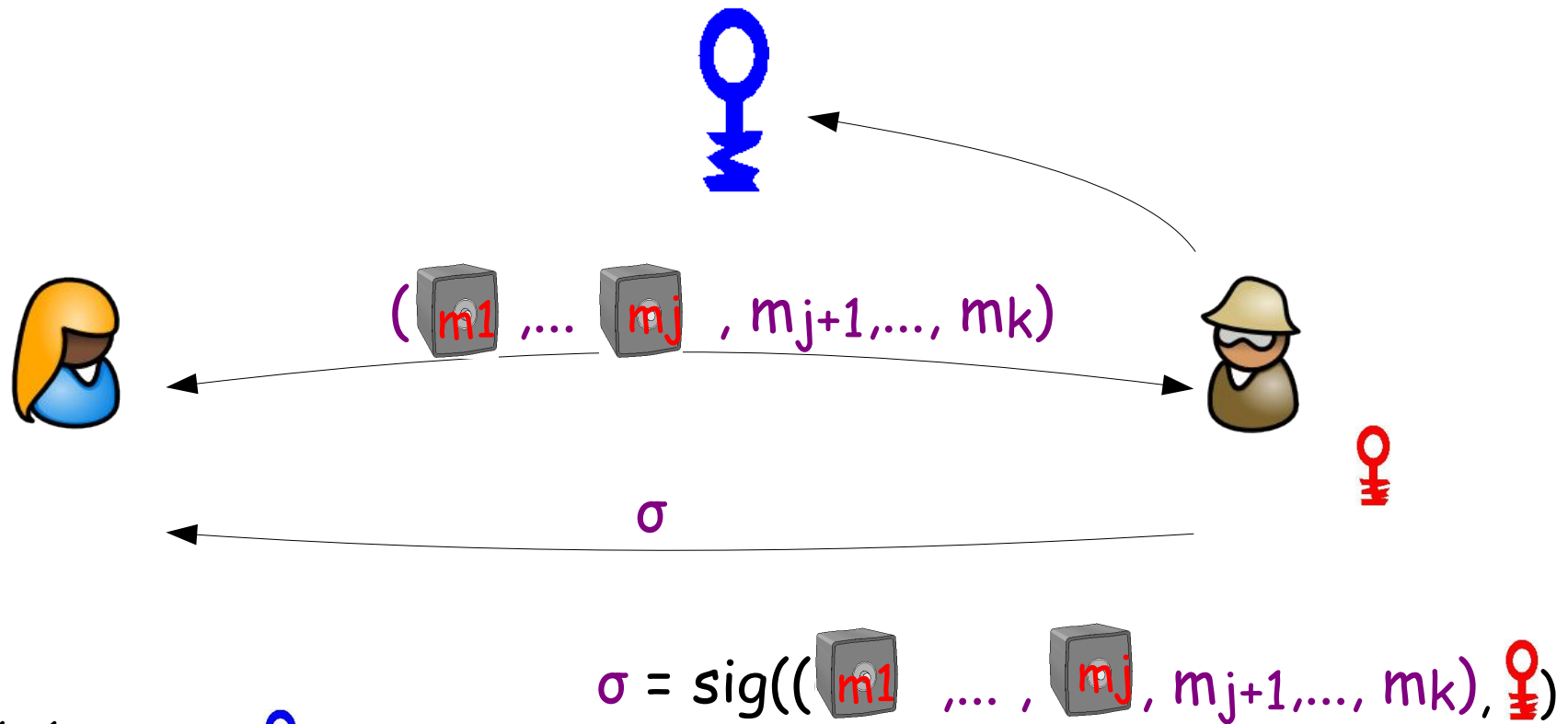


Problem: Pseudonym not in message space!

Solution: Sign secret key instead

$$\rightarrow d = c^e a_1^{sk} \cdot a_2^{m_2} \dots a_k^{m_k} b^s \bmod n$$

New Problem: how can we sign a secret message?




$\text{ver}(\sigma, (m_1, \dots, m_k), \text{public key}) = \text{true}$

$$\sigma = \text{sig}((m_1, \dots, m_j, m_{j+1}, \dots, m_k), \text{private key})$$

Verification remains unchanged!

Security requirements basically the same as for signatures, but

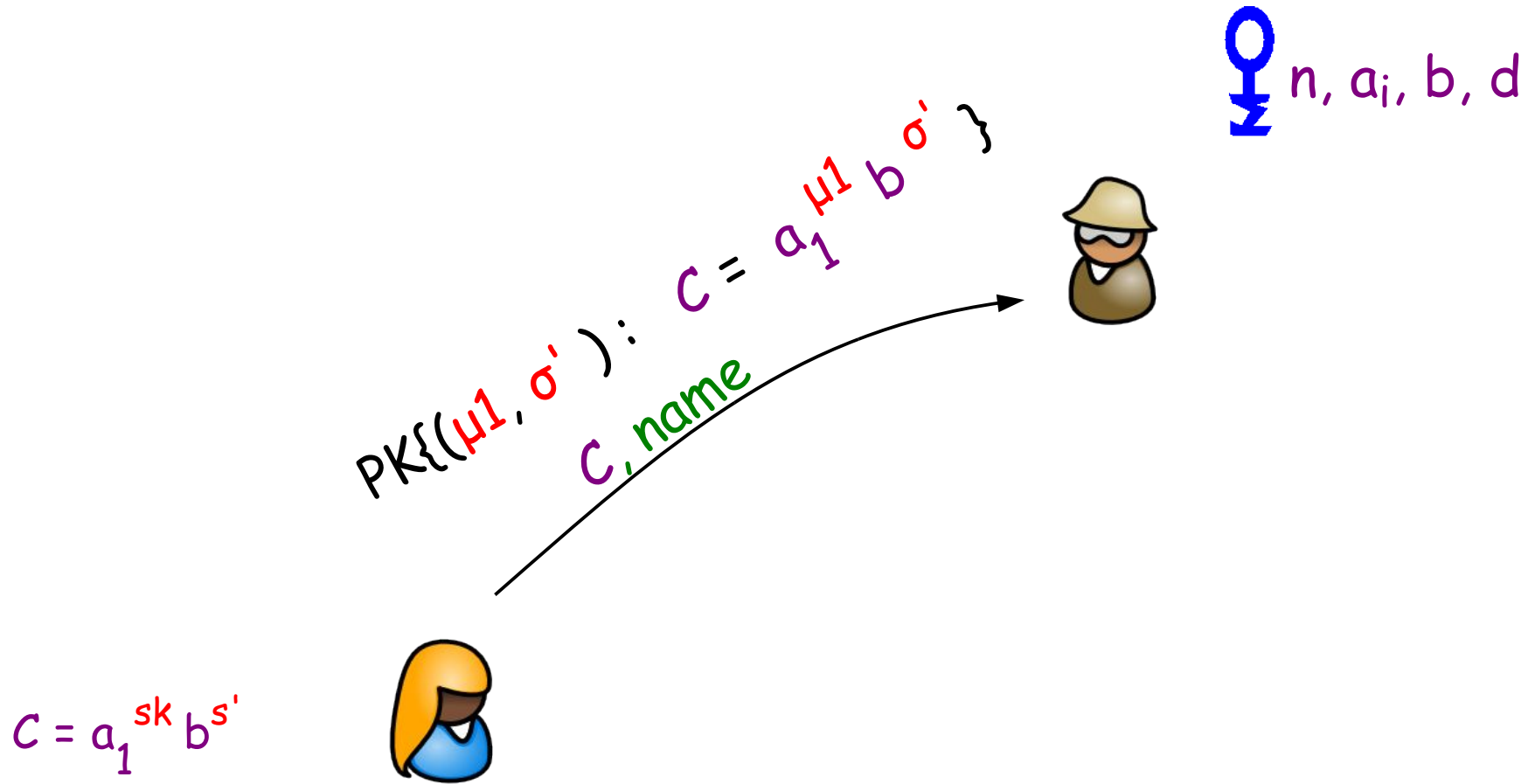
- signer should not learn any information about  $m_1, \dots, m_j$
- Forgery w.r.t. message clear parts and opening of commitments

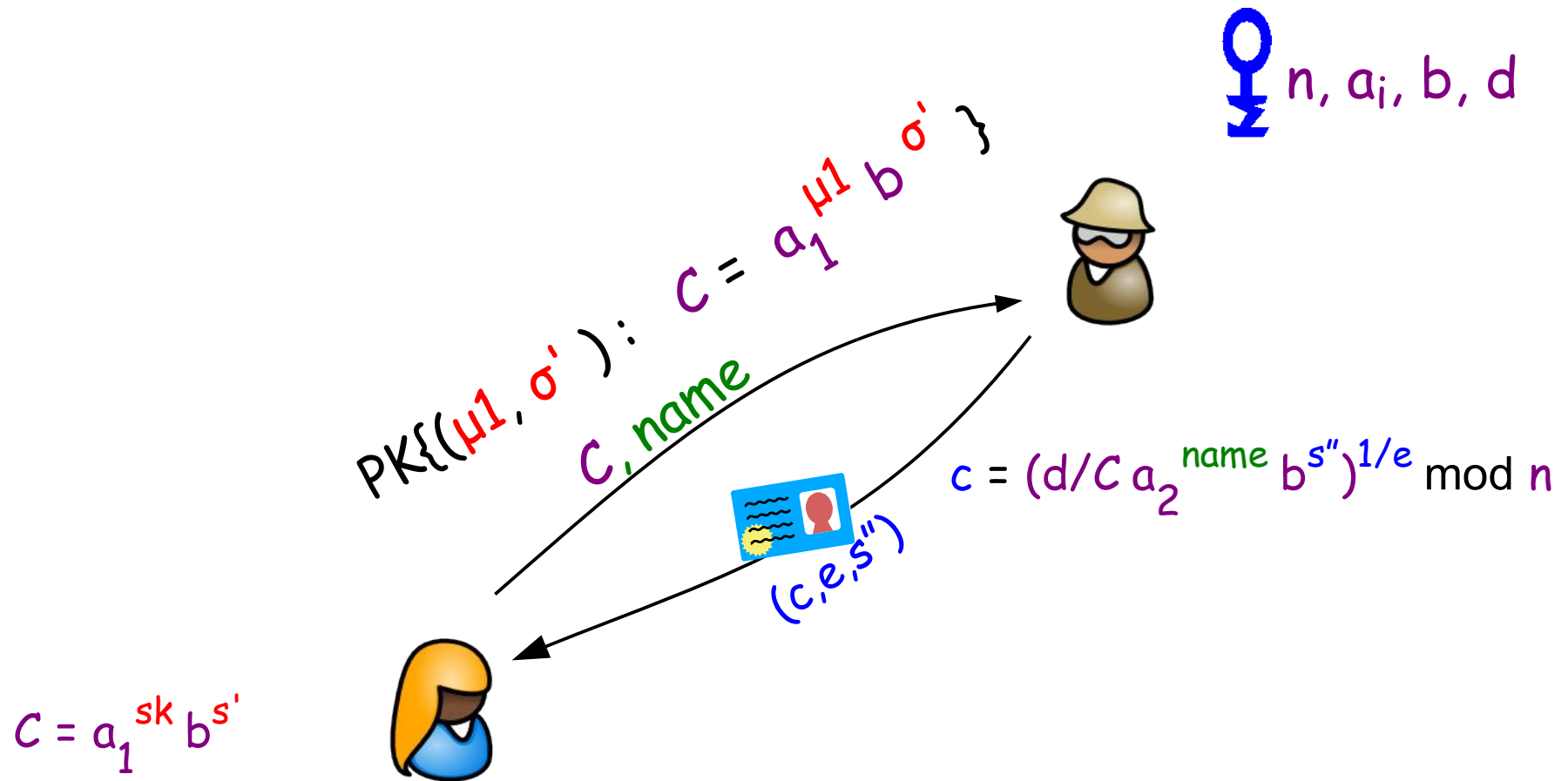
  $n, a_i, b, d$

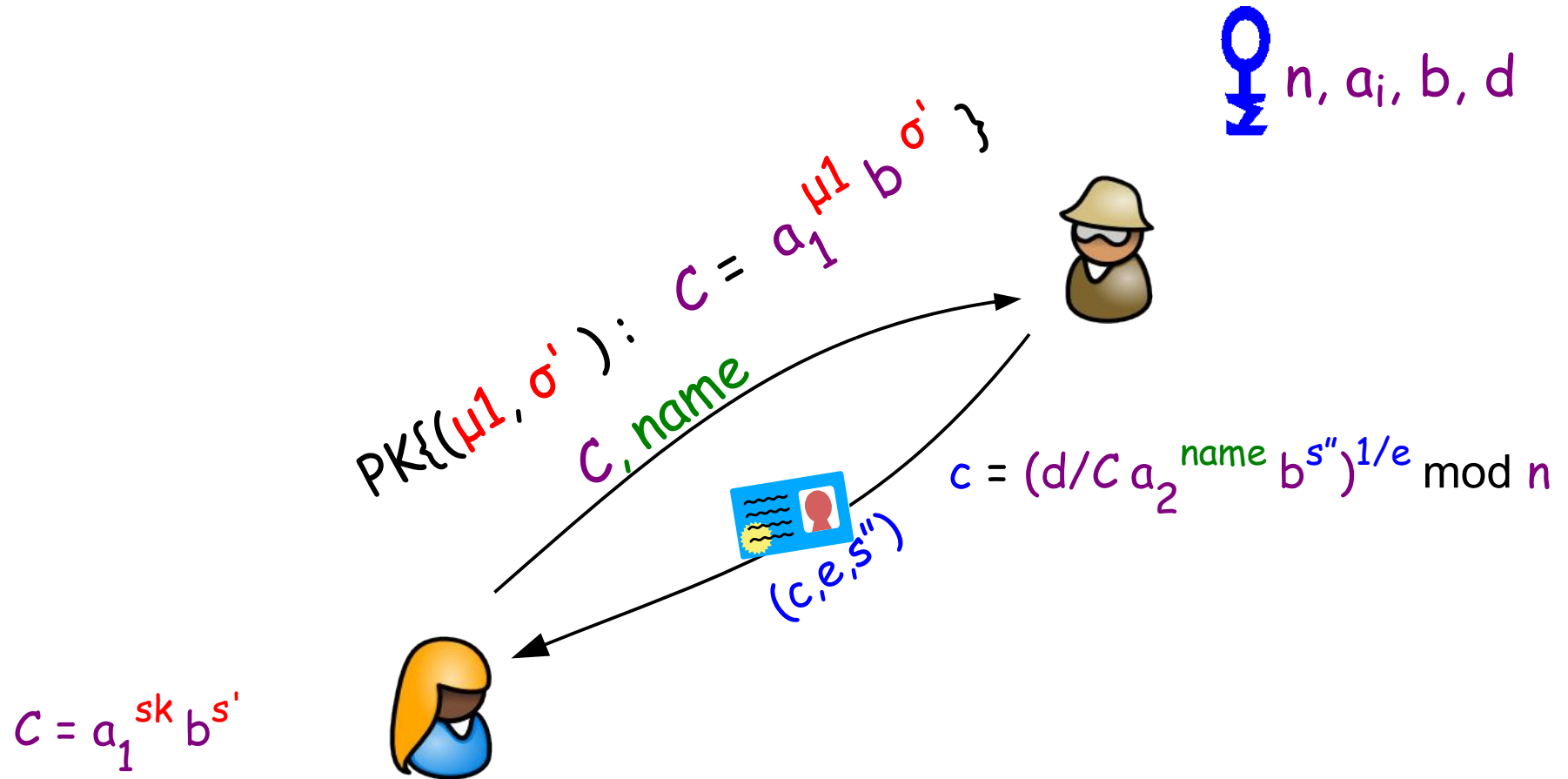


$$C = a_1^{sk} b^{s'}$$





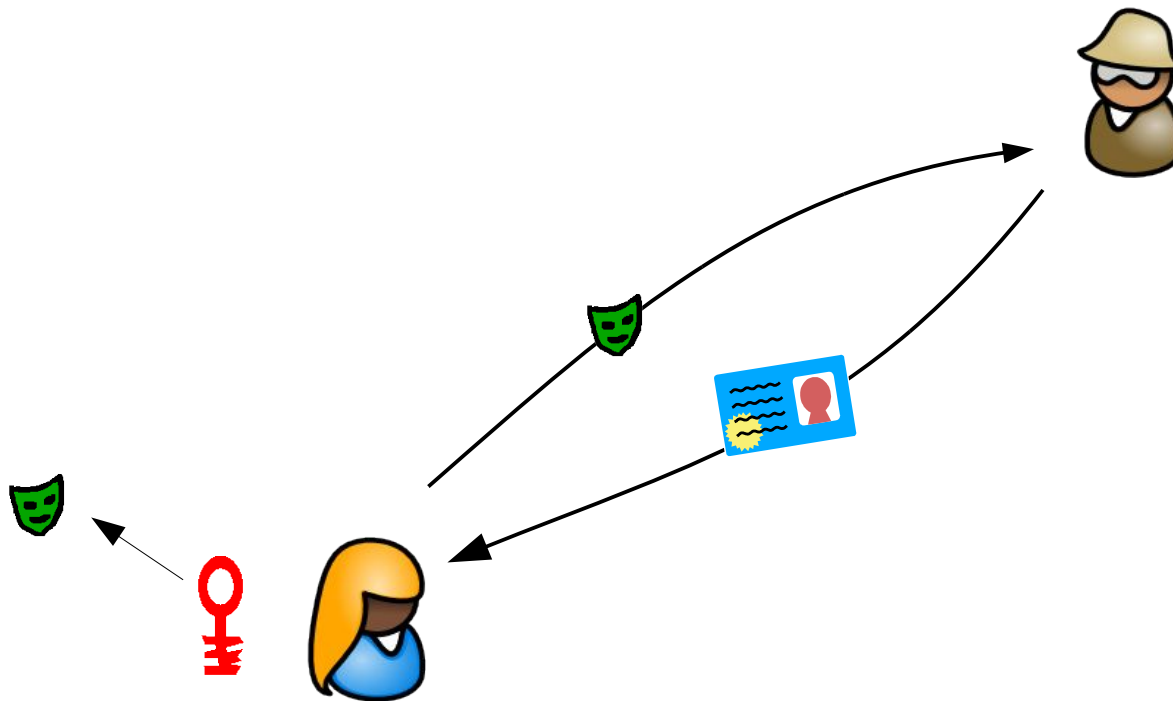




$$d = c^e a_1^{sk} a_2^{name} b^{s'' + s'} \pmod{n}$$

Want to sign w.r.t.  $Nym = g^{sk} h^r \text{mask}$

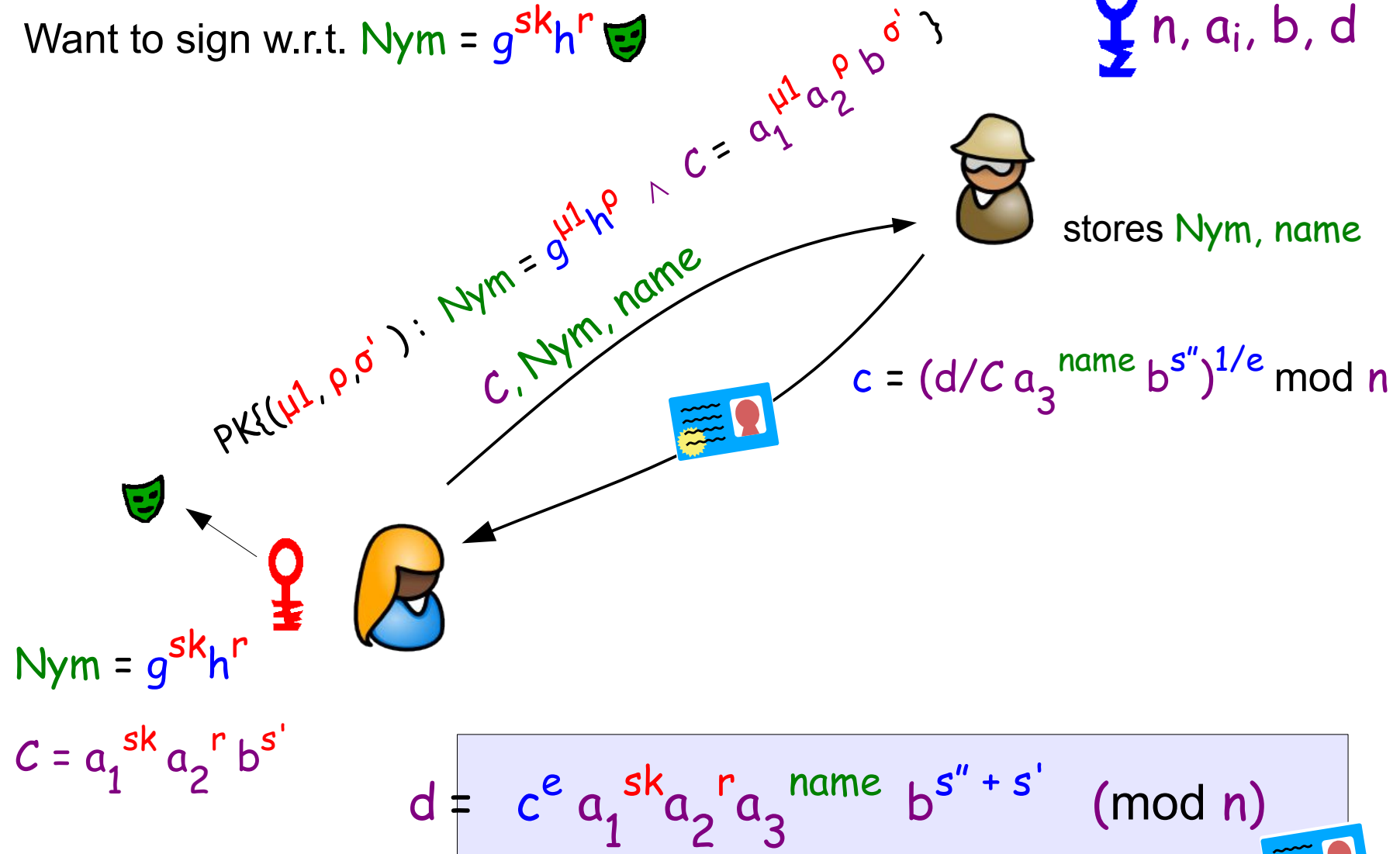
$\Sigma n, a_i, b, d$





Want to sign w.r.t.  $Nym = g^{sk} h^r$

$\Sigma n, a_i, b, d$

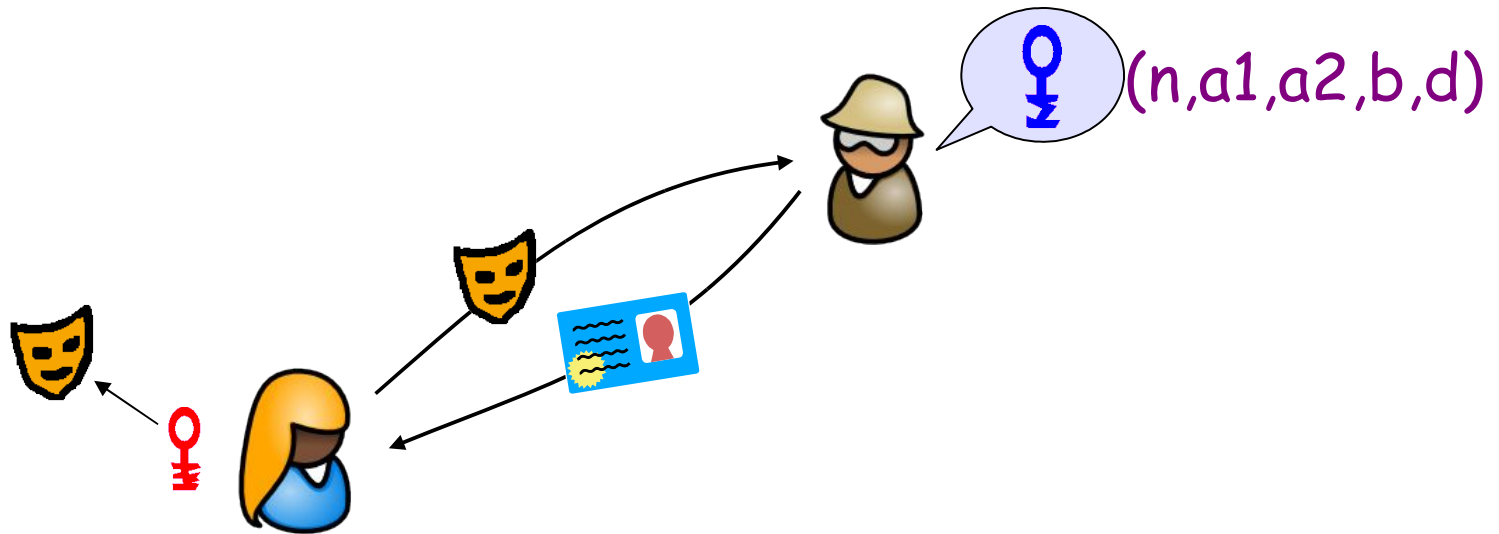


A close-up photograph of a sandy surface. In the upper right, there is a small, dried, brown leaf fragment. Below it, there are several dark, circular indentations or tracks in the sand, arranged in a somewhat vertical line. The sand is a light brown color with some darker spots and small pebbles.

# An Example Scenario

## Scenario:

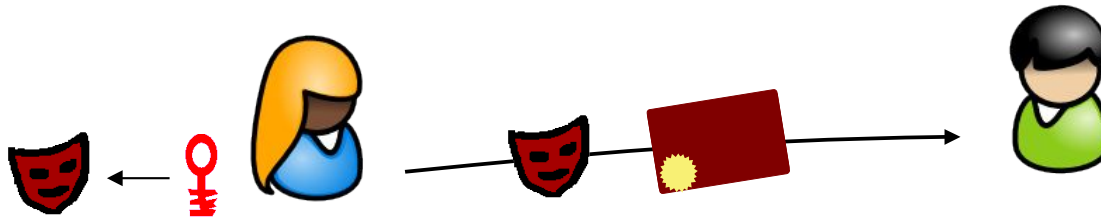
- Pollster(s) and a number of users
- Only registered user (e.g., students who took a course) can voice opinion (e.g., course evaluation)
- User can voice opinion only once (subsequent attempts are dropped)
- Users want to be anonymous
- A user's opinion in different polls must not be linkable


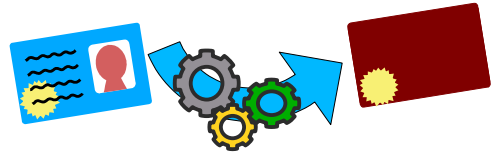


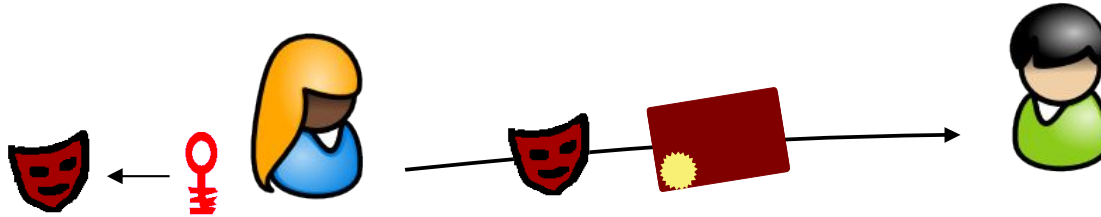
- User generates pseudonym (ID for registration)
- User obtains credential on pseudonym stating that she is eligible for polls, i.e.,  $(c, e, s)$

$$d = c^e a_1^{sk} a_2^r a_3^{attr} b^s \pmod{n}$$

- Credential can contain attributes (e.g., course ID) about her



1. User generates domain pseudonym, domain = pollID 
2. User transforms credential 
3. Transformed credential with a subset of the attributes
  - User is anonymous and unlinkable
  - Multiple opinions are detected because uniqueness of domain pseudonym



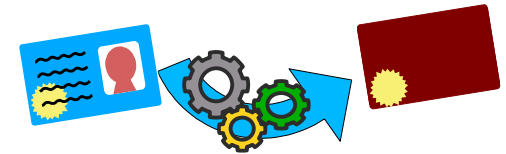
1. Domain pseudonym:  $P = g_d^{sk} = H(\text{pollID})^{sk}$

$P1 = H(\text{pollID1})^{sk}$  and  $P2 = H(\text{pollID2})^{sk}$  are unlinakble  
(under the Decisional Diffie-Hellman assumption)



2. User transforms credential:

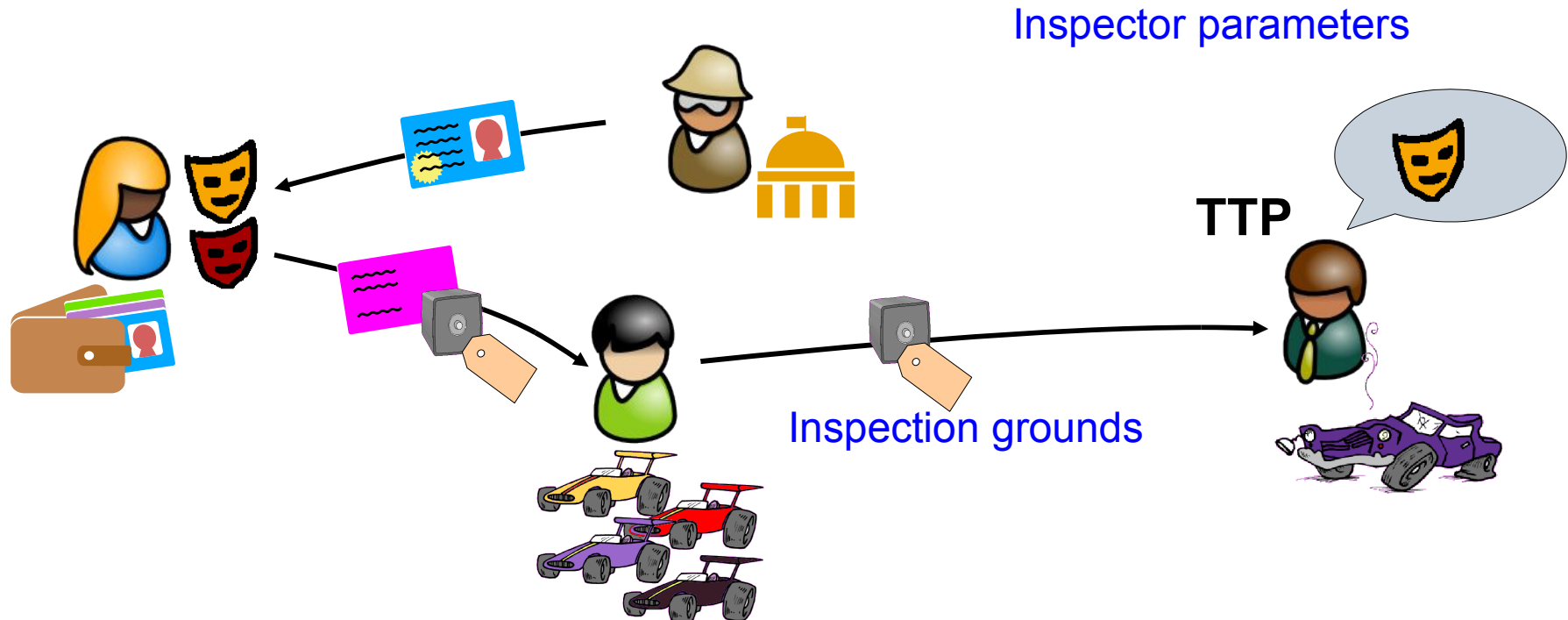
- $c' = c b^{s'} \text{ mod } n$  with randomly chosen  $s'$
- $\text{SPK}\{(\epsilon, \mu1, \mu2, \mu3, \sigma) : P = g_d^{\mu1} \wedge d := c'^{\epsilon} a1^{\mu1} a2^{\mu2} a3^{\mu3} b^{\sigma} \text{ (mod } n) \wedge \mu1, \mu2, \mu3 \in \{0,1\}^{\ell} \wedge \epsilon > 2^{\ell+1} \}(\text{opinion})$



A photograph of a beach at sunset or sunrise. The ocean waves are visible in the upper left, with a white foam line. The sand is dark and textured. A single, dark footprint is visible in the foreground, centered horizontally and slightly below the middle vertically. The text "Further Concepts" is overlaid in white, sans-serif font on the left side of the image.

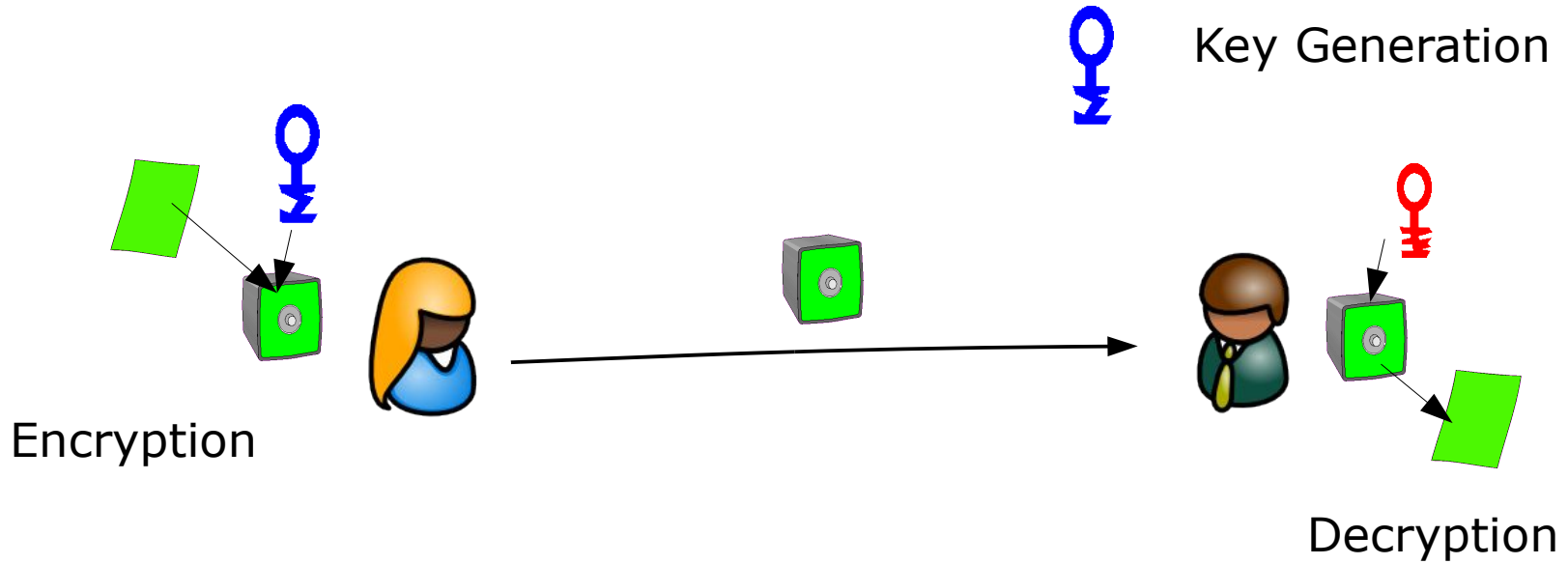
# Further Concepts



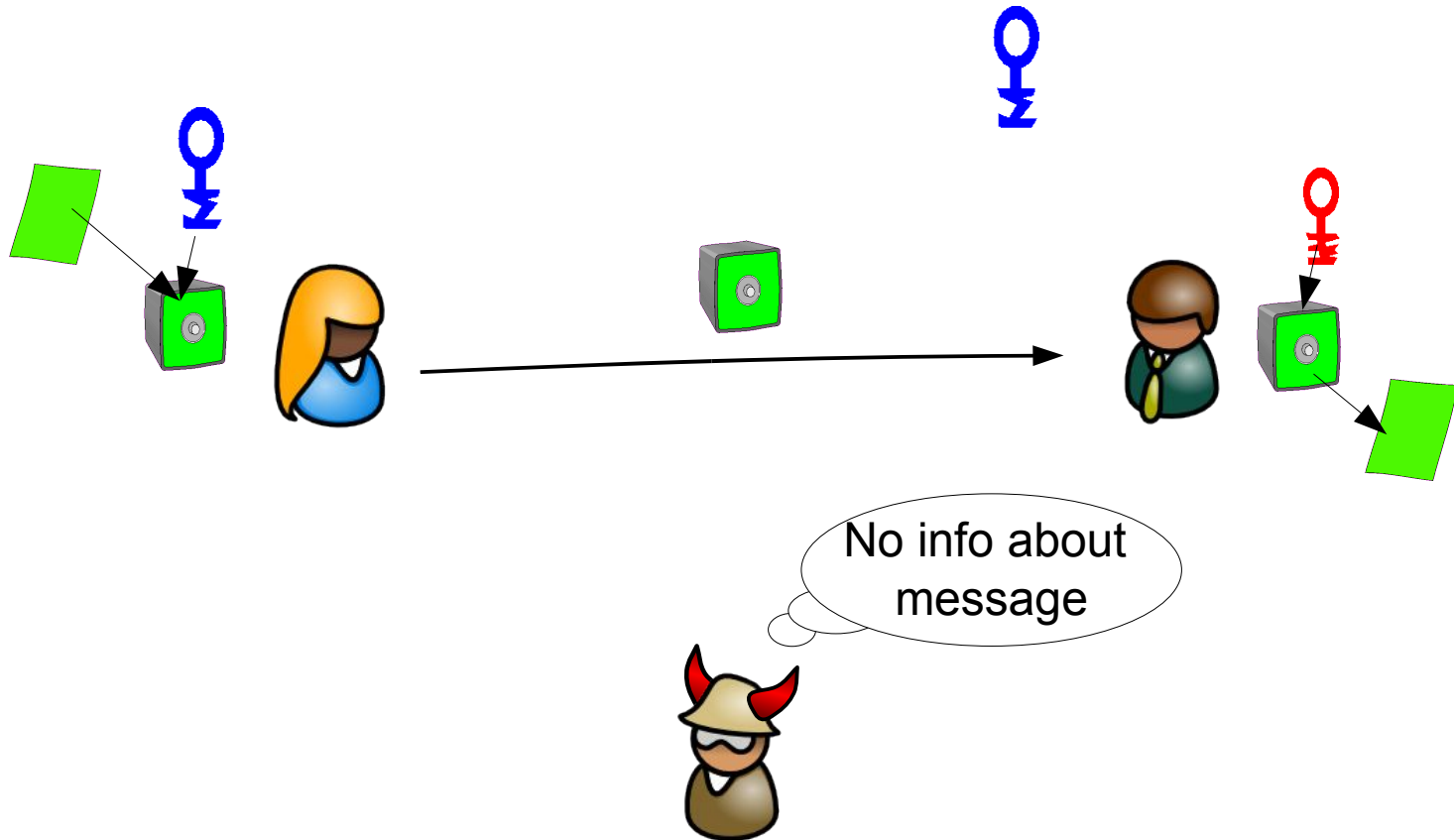


- If car is damaged: ID with insurance or gov't needs be retrieved
- Similarly: verifiably encrypt any certified attribute (*optional*)
- TTP is off-line & can be distributed to lessen trust

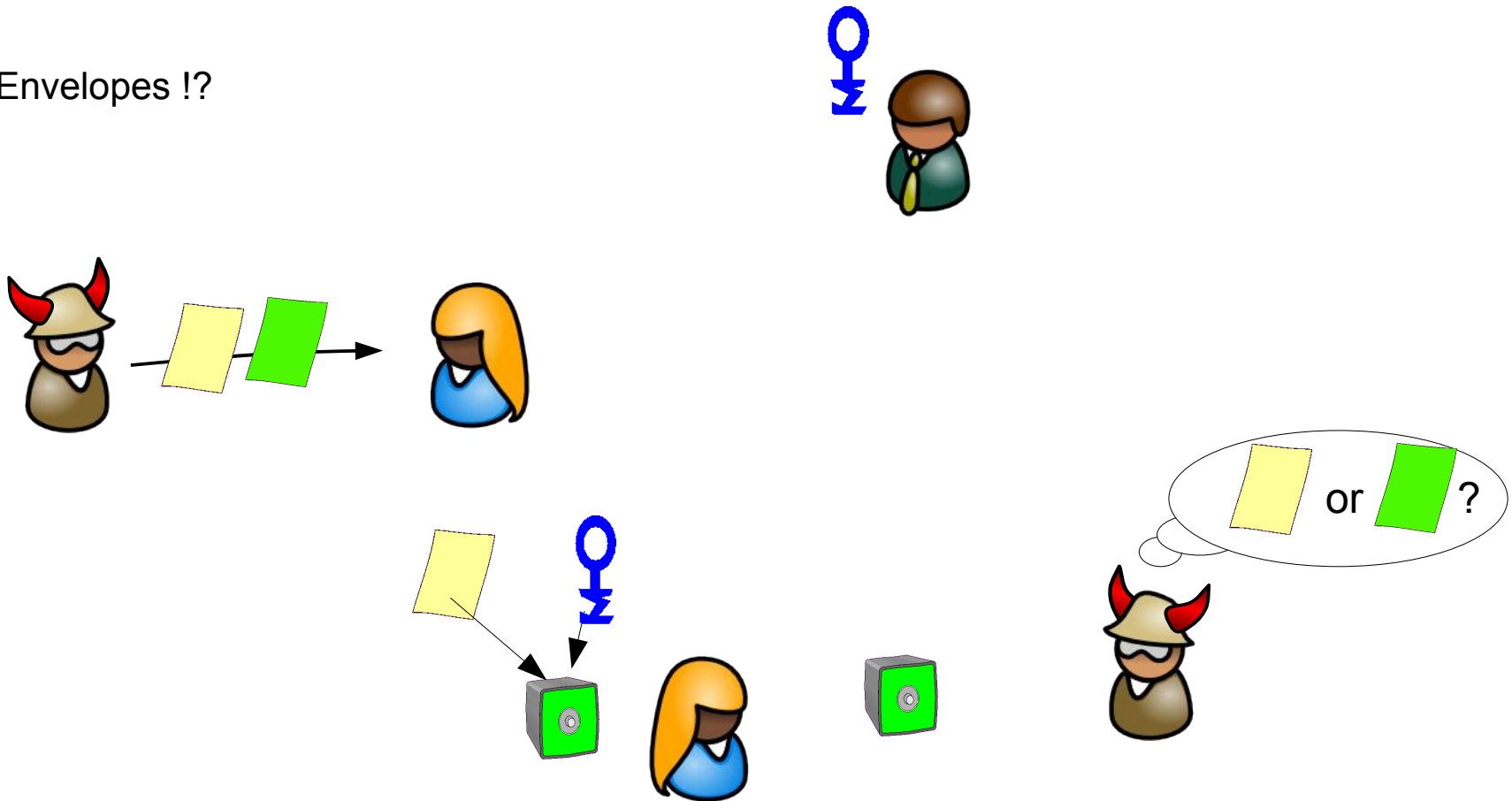




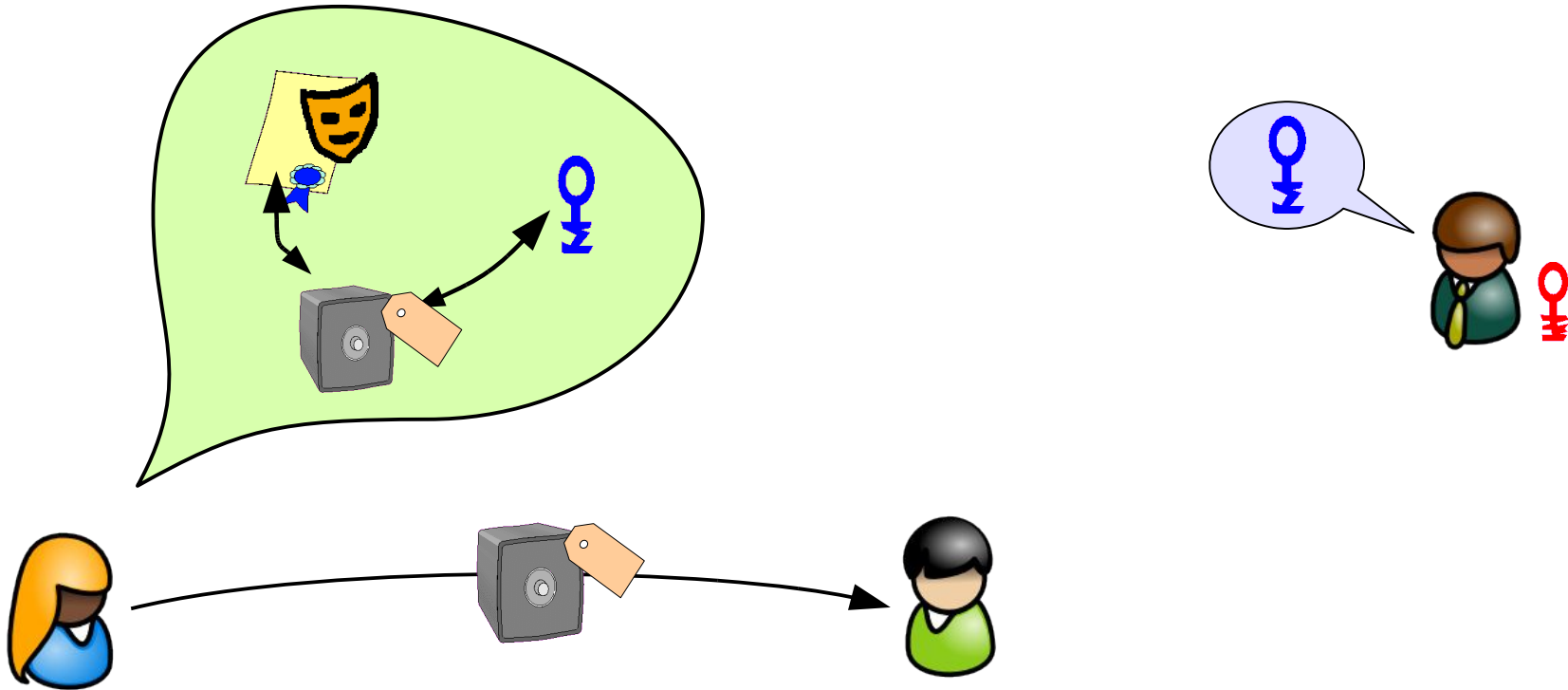
Like Envelopes !?



Like Envelopes !?



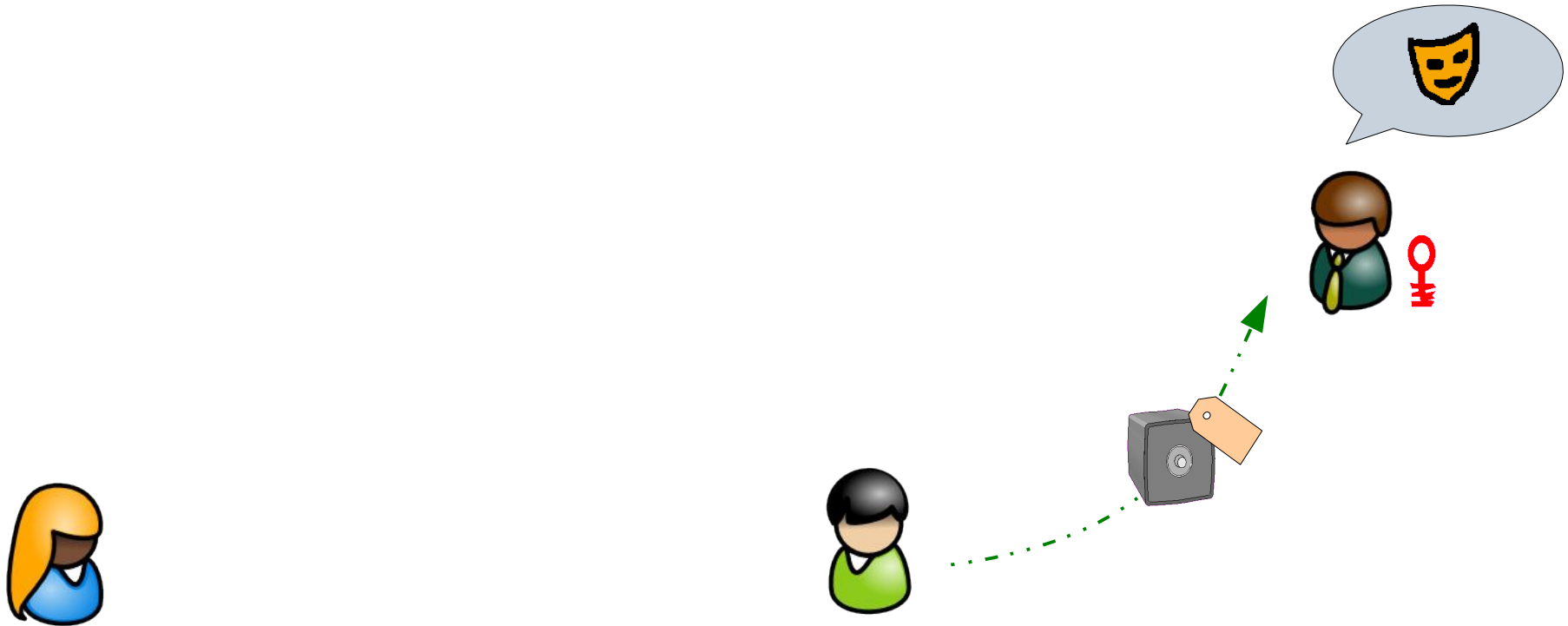
This is called *semantic security* (secure if used once only or within careful construction.)



Label is important to bind context to an encryption.

E.g., defines decryption condition, binds user to car, etc.

Security definition: change of label is new ciphertext



- Of attributes (discrete logarithm)
  - Camenisch-Shoup (SRSA) – based on Paillier Encryption
- Of pseudonyms (group elements)
  - Cramer-Shoup (DL) or rarely ElGamal (DL)
- Otherwise (any secret for which ZKPK exists)
  - Camenisch-Damgaard, works for any scheme, but much less efficient
- ....Open Problem to find new ones!

- Group  $G = \langle g \rangle$  of order  $q$
- Secret Key Group  $x \in \{1, \dots, q\}$ ; Public key  $y = g^x$
- To encrypt message  $m \in G$ :
  - choose random  $r \in \{1, \dots, q\}$ ;
  - compute  $c = (y^r m, g^r)$
- To decrypt ciphertext  $c = (c_1, c_2)$ 
  - We know  $c = (y^r m, g^r) = (g^{xr} m, g^r)$
  - Thus set  $m = c_1 c_2^{-x} = y^r m g^{-xr} = y^{r-r} m = m$



Nym

$$\text{Nym} = g^{\text{sk}} h^r$$

$$d = c^e a_1^{\text{sk}} a_2^r a_3^{\text{name}} b^{s'' + s'} \pmod{n}$$



$$y = g^x$$

- Encrypt **Nym** : random  $u \in \{1, \dots, q\}$  and  $\text{enc} = (y^u \text{Nym}, g^u) = (e1, e2)$
- Compute proof token (presentation token):
  - compute  $c' = c b^{\dagger} \pmod{n}$  with randomly chosen  $\dagger$
  - compute proof

PK $\{(\epsilon, \mu1, \mu2, \mu3, \sigma) :$

$$d := c'^{\epsilon} a_1^{\mu1} a_2^{\mu2} a_3^{\mu3} b^{\sigma} \wedge e1 = y^{\rho} g^{\mu1} h^{\mu2} \wedge e2 = g^{\rho} \wedge$$

$$\mu1, \mu2, \mu3 \in \{0,1\}^{\ell} \wedge \epsilon > 2^{\ell+1} \}$$

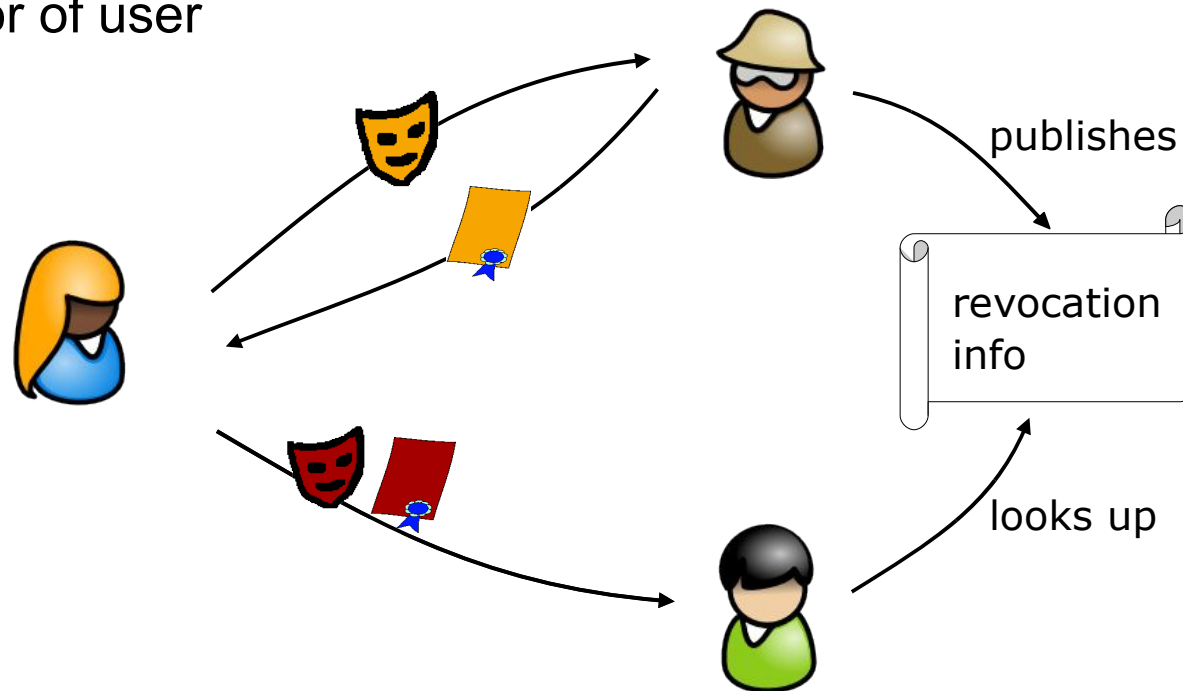


A background image of a sandy beach with two footprints. One footprint is in the upper middle, and another is in the lower left. The text 'Revocation of credentials' is overlaid in the center.

Revocation of credentials

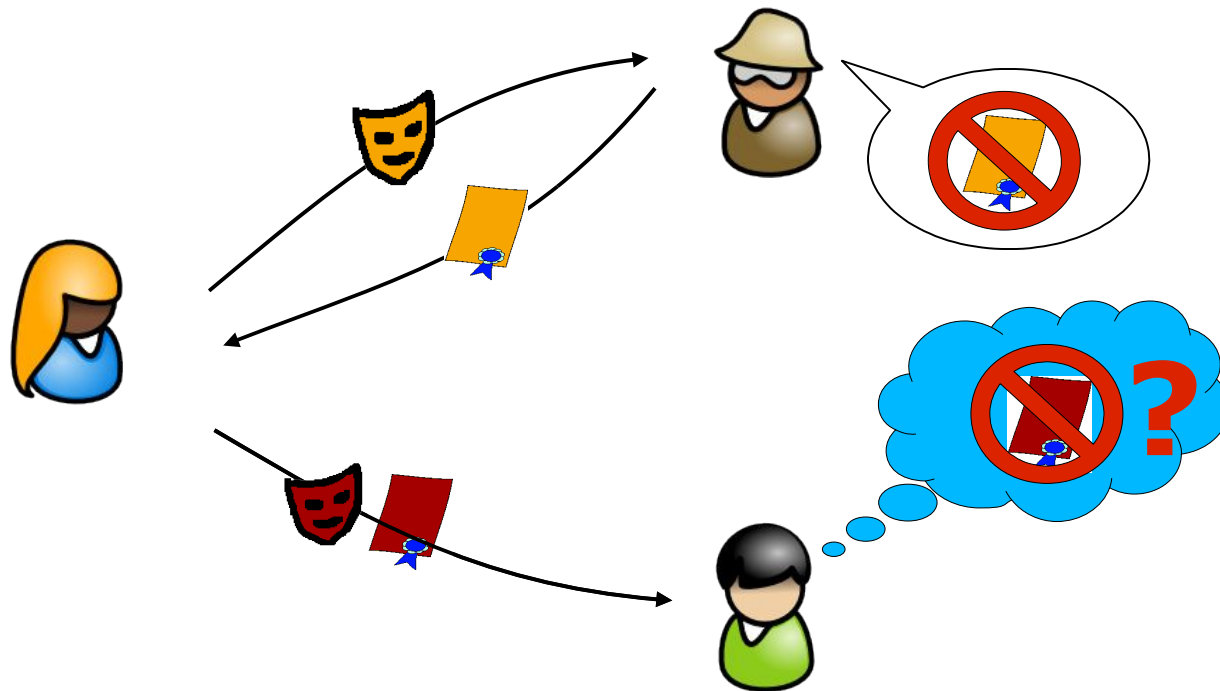
various reasons to revoke credential

- user lost credential / secret key
- misbehavior of user



Alice should be able to convince verifier that her credential is among the good ones!

- Pseudonyms → standard revocation lists don't work



- Include into credential some credential ID  $ui$  as message, e.g.,

$$d = c^e a_1^{sk} a_2^{ui} b^{s'' + s'} \pmod{n}$$

- Publish list of all valid (or invalid)  $ui$ 's.

$$(u1, \dots, uk)$$

- Alice proves that her  $ui$  is on the list.

- Choose random  $g$

- Compute  $Uj = g^{uj}$  for  $uj$  in  $(u1, \dots, uk)$

- Prove  $PK\{(\epsilon, \mu, \rho, \sigma) : (d = c'^{\epsilon} a_1^{\rho} a_2^{\mu} b^{\sigma} \pmod{n}) \wedge U1 = g^{\mu}\}$

$$\vee \dots \vee (d = c'^{\epsilon} a_1^{\rho} a_2^{\mu} b^{\sigma} \pmod{n}) \wedge Uk = g^{\mu} ) \}$$

- Not very efficient, i.e., linear in size  $k$  of list :-)

- Include into credential some credential ID  $ui$  as message, e.g.,

$$d = c^e a_1^{sk} a_2^{ui} b^{s'' + s'} \pmod{n}$$

- Publish list of all *invalid*  $ui$ 's.

$$(u_1, \dots, u_k)$$

- Alice proves that her  $ui$  is not on the list.

– Choose random  $h$  and compute  $U = h^{ui}$

– Prove  $PK\{(\epsilon, \mu, \rho, \sigma) : d = c'^{\epsilon} a_1^{\rho} a_2^{\mu} b^{\sigma} \pmod{n}$

$$\wedge U = h^{\mu} \}$$

– Verifier checks whether  $U = h^{u_j}$  for all  $u_j$  on the list.

- Better, as *only verifier* needs to do linear work (and it can be improved using so-call batch-verification...)
- What happens if we make the list of all valid  $ui$ 's public?
- If credential is revoked, all past transactions become linkable...

Variation: verifier could choose  $h$  and keep it fixed for a while

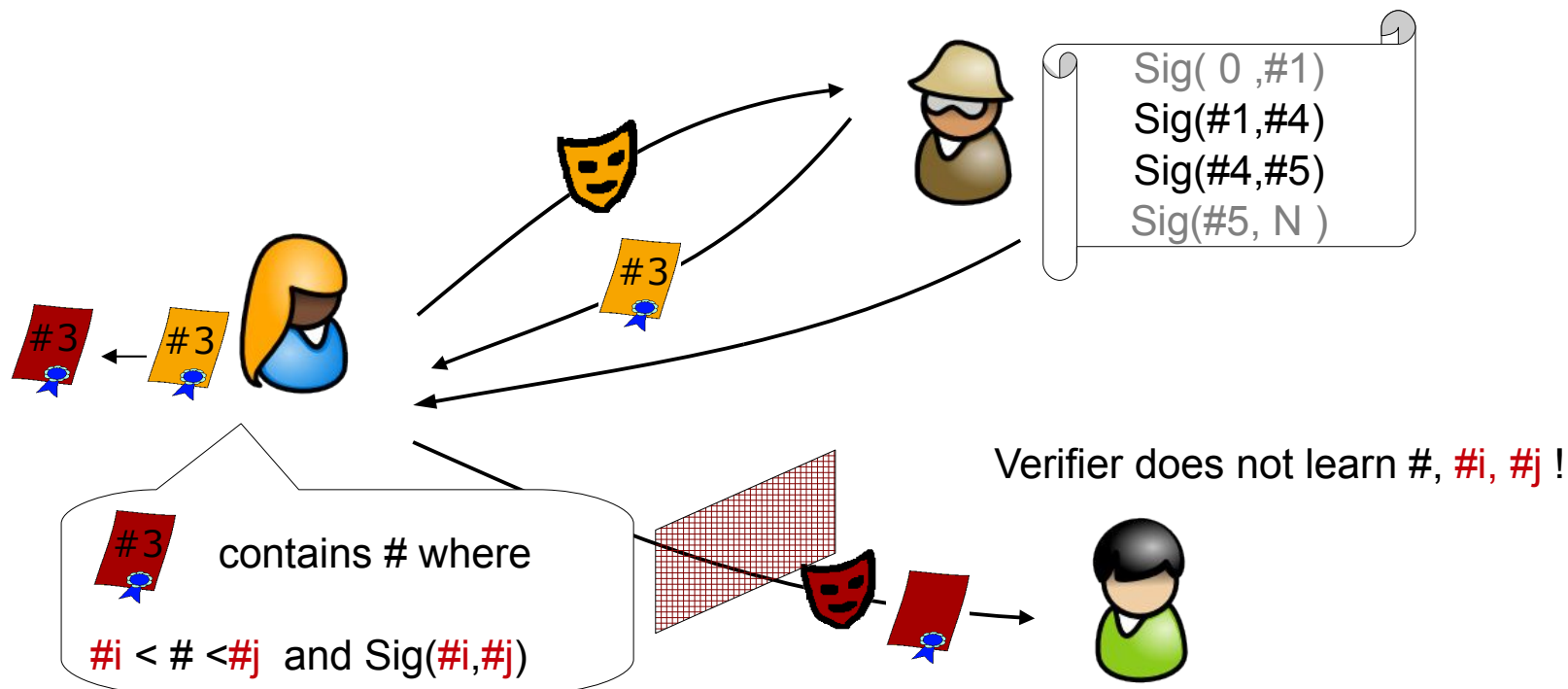
- Can pre-compute list  $U_i = h^{u_i}$
- $\rightarrow$  single table lookup
- BUT: if user comes again, verifier can link!!!
- ALSO: verifier could not change  $h$  at all! or use the same as other verifiers!
  - one way out  $h = H(\text{verifier}, \text{date})$ , so user can check correctness.
  - $\text{date}$  could be the time up to seconds and the verifier could just store all the lists, i.e., pre-compute it.



... better implementation of proof :

Issuer signs intervals between revoked #

→ revocation list: #1, #4, #5,

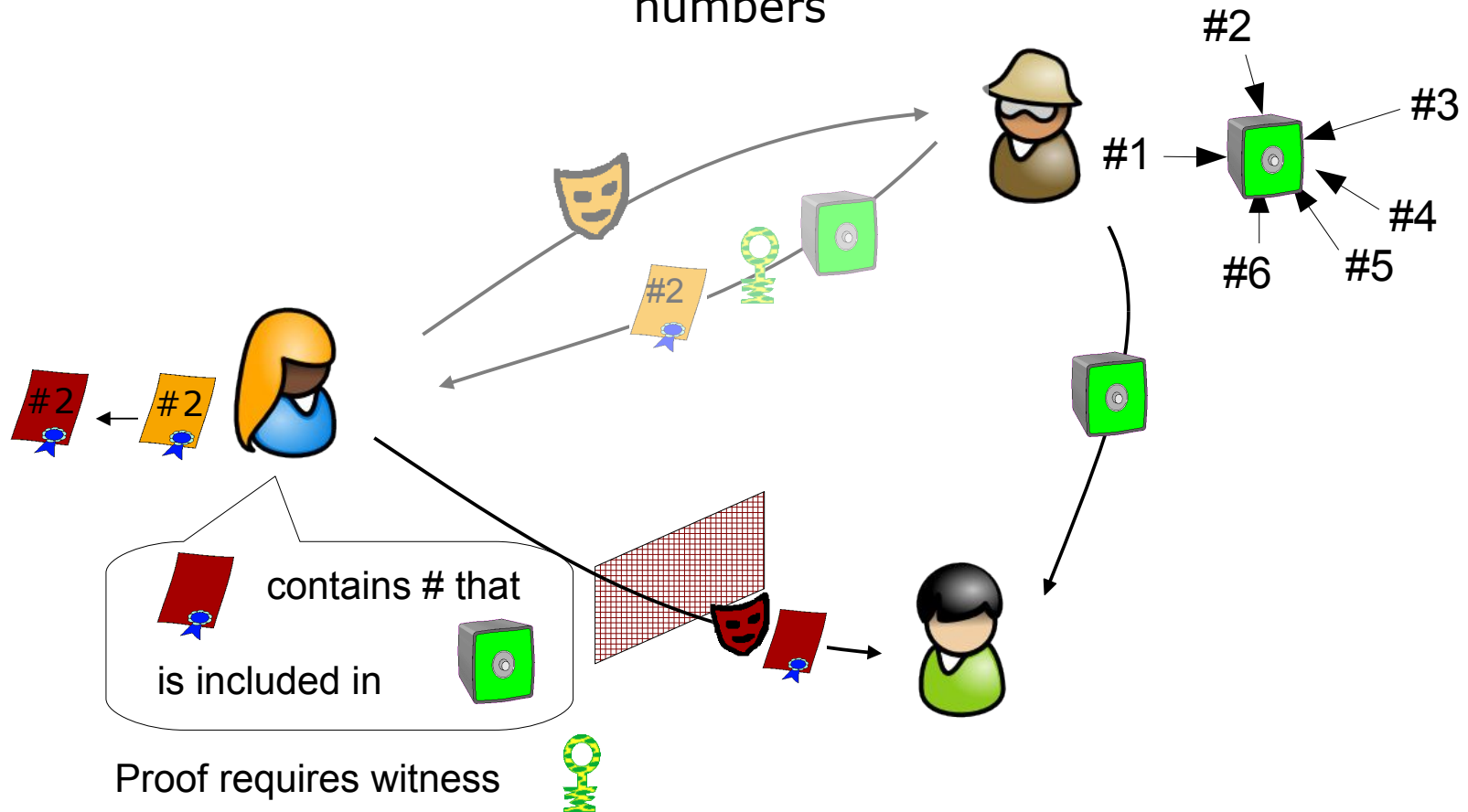


$\text{Sig}(\#i, \#j)$  can be realised also with credential signature scheme, using different public key

Using **cryptographic accumulators**:

credentials contain random serial number #

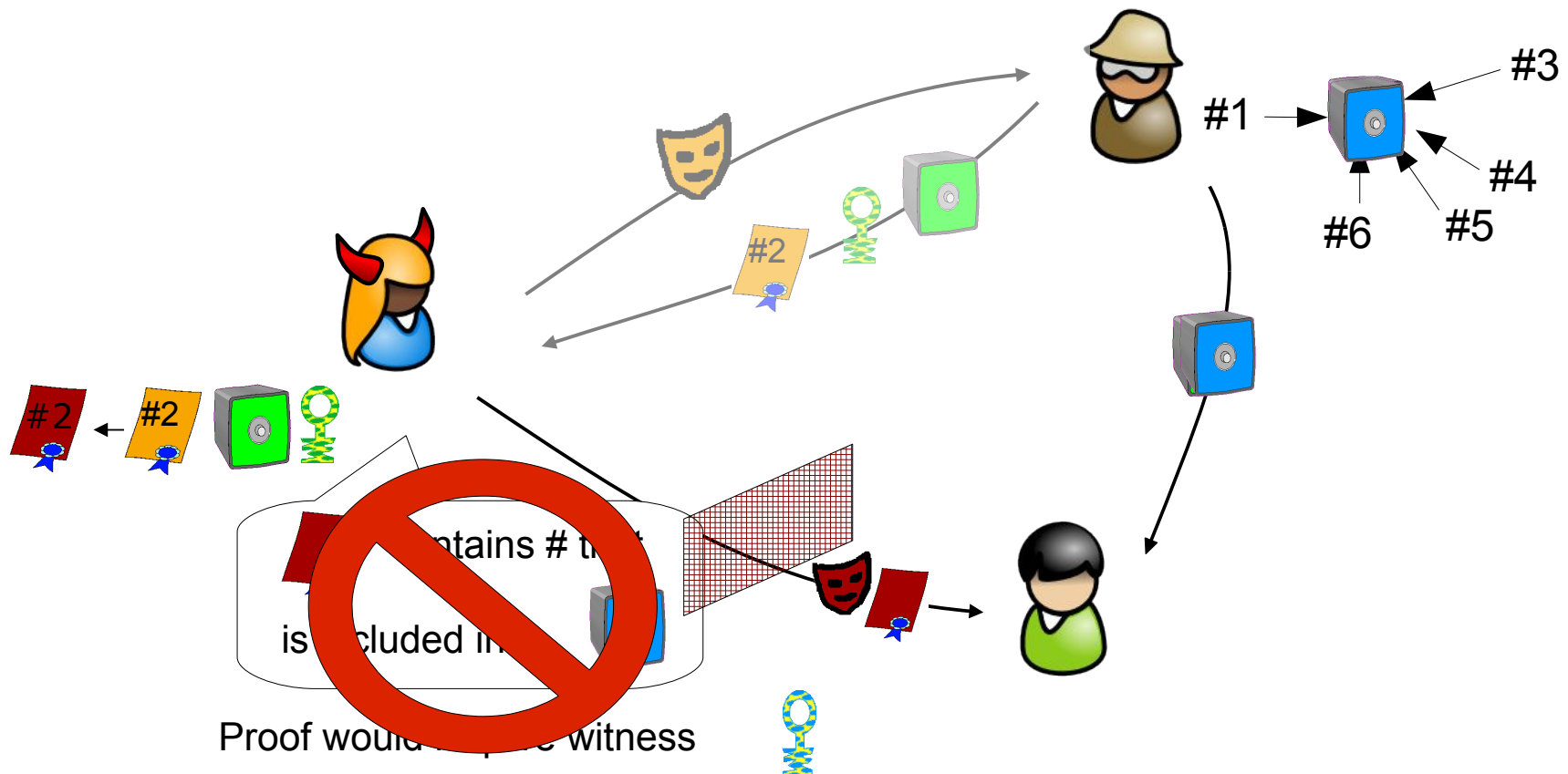
Issuer accumulates all "good" serial numbers





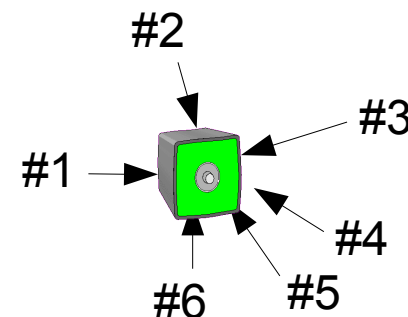
Using cryptographic accumulators:

to revoke #2 issuer publishes new accumulator & new witnesses for unrevoked credentials



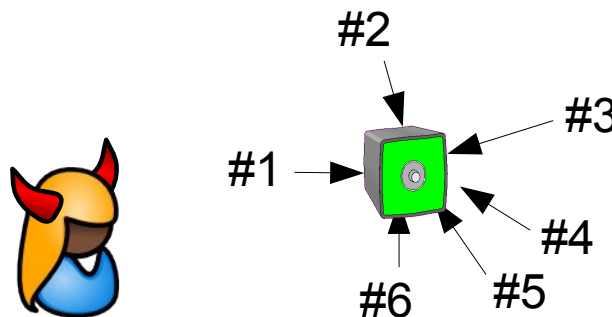
Using so-called cryptographic accumulators:

- Key setup: RSA modulus  $n$ , seed  $v$
- Accumulate:
  - values are primes  $e_i$
  - accumulator value:  $z = v^{\prod e_i} \bmod n$
  - publish  $z$  and  $n$
  - witness value  $x$  for  $e_j$  : s.t.  $z = x^{e_j} \bmod n$   
can be computed as  $x = v^{e_1 \dots e_{j-1} \cdot e_{j+1} \dots e_k} \bmod n$
- Show that your value  $e$  is contained in accumulator:
  - provide  $x$  for  $e$
  - verifier checks  $z = x^e \bmod n$



Security of accumulator: show that  $e$  s.t.  $z = x^e \bmod n$  for  $e$  that is not contained in accumulator:

- For fixed  $e$ : Equivalent to RSA assumption
- Any  $e$ : Equivalent to Strong RSA assumption



Revocation: Each cert is associated with an  $e$  and each user gets witness  $x$  with certificate. But we still need:

- Efficient protocol to prove that committed value is contained in accumulator.
- Dynamic accumulator, i.e., ability to remove and add values to accumulator as certificates come and go.

- Prove that your key is in accumulator:

- Commit to  $x$ :

- choose random  $s$  and  $g$  and

- compute  $U1 = x h^s$ ,  $U2 = g^s$  and reveal  $U1, U2, g$

- Run proof-protocol with verifier

- PK $\{(\epsilon, \mu, \rho, \sigma, \xi, \delta) :$

$$d = c'^{\epsilon} a_1^{\rho} a_2^{\mu} b^{\sigma} \pmod{n} \wedge z = U1^{\mu} (1/h)^{\xi} \pmod{n}$$

$$\wedge 1 = U2^{\mu} (1/g)^{\xi} \pmod{n} \wedge U2 = g^{\delta} \pmod{n}$$

## ■ Analysis

–No information about  $x$  and  $e$  is revealed:

- $(U1, U2)$  is a secure commitment to  $x$
- proof-protocol is zero-knowledge

–Proof is indeed proving that  $e$  contained in the certificate is also contained in the accumulator:

$$\text{a) } 1 = U2^\mu (1/g)^\xi = (g^\delta)^\mu (1/g)^\xi \pmod{n}$$

$$\Rightarrow \xi = \delta \mu$$

$$\text{b) } z = U1^\mu (1/h)^\xi = U1^\mu (1/h)^{\delta \mu} = (U1/h^\delta)^\mu \pmod{n}$$

$$\text{c) } d = c'^\epsilon a_1^\rho a_2^\mu b^\sigma \pmod{n}$$

## Dynamic Accumulator

- When a new user gets a certificate containing  $e_{\text{new}}$ 
  - Recall:  $z = v^{\prod e_i} \bmod n$
  - Thus:  $z' = z^{e_{\text{new}}} \bmod n$
  - But: then all witnesses are no longer valid, i.e., need to be updated  $x' = x^{e_{\text{new}}} \bmod n$

## Dynamic Accumulator

- When a certificate containing  $e_{rev}$  revoked

– Now  $z' = v^{\prod e_i} = z^{1/e_{rev}} \bmod n$

– Witness:

- Use Ext. Euclid to compute  $a$  and  $b$

$$\text{s.t. } a e_{own} + b e_{rev} = 1$$

- Now  $x' = x^b z'^a \bmod n$

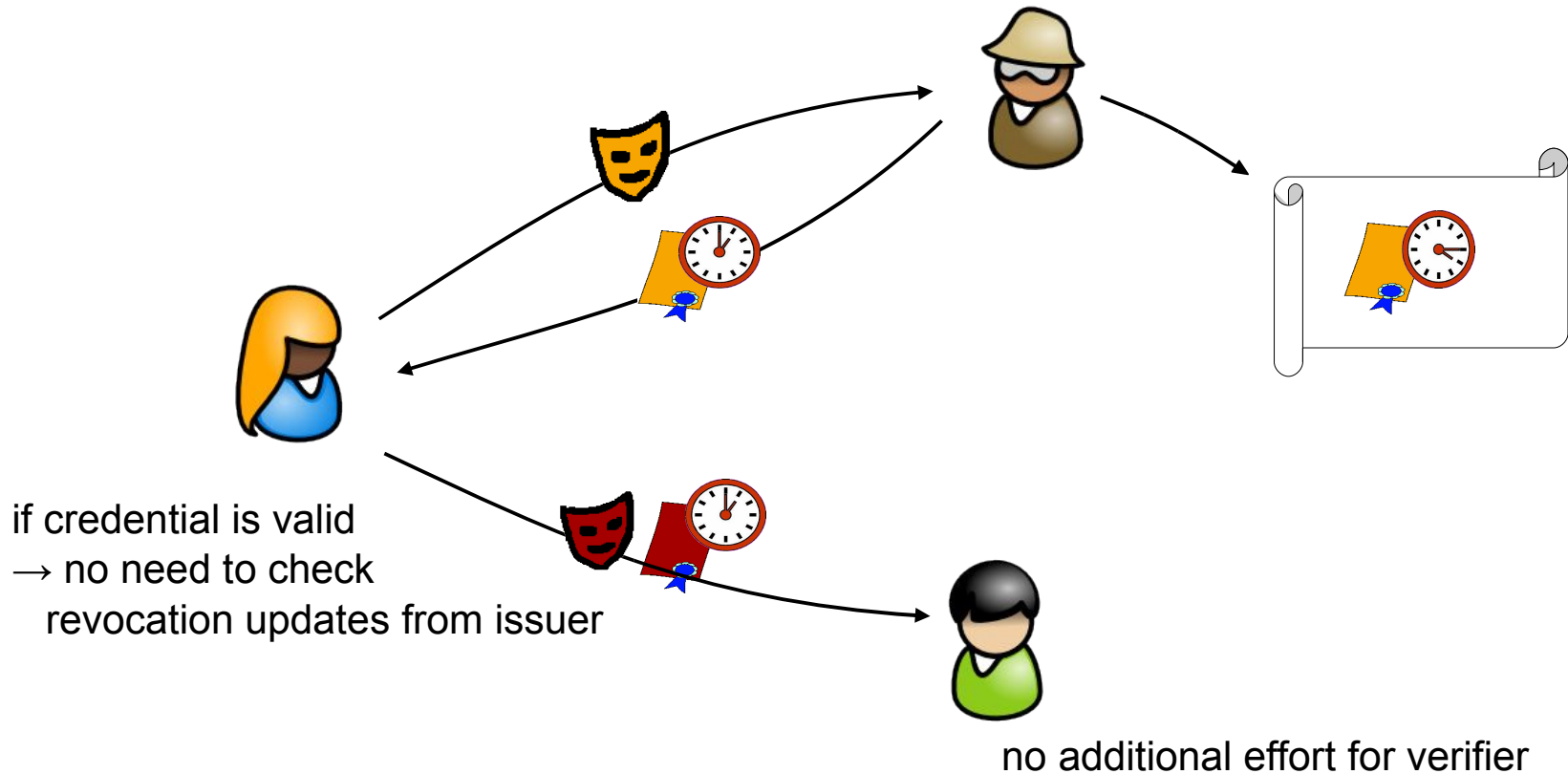
- Why:  $x'^{e_{own}} = ((x^b z'^a)^{e_{own}})^{e_{rev} 1/e_{rev}} \bmod n$   
 $= ((x^b z'^a)^{e_{own} e_{rev}})^{1/e_{rev}} \bmod n$   
 $= ((x^{e_{own}})^b e_{rev} (z'^{e_{rev}})^a e_{own})^{1/e_{rev}} \bmod n$   
 $= (z^{b e_{rev}} z^{a e_{own}})^{1/e_{rev}} \bmod n$   
 $= z^{1/e_{rev}} \bmod n = z' \quad :-)$

Dynamic Accumulator: in case the issuer knows the factorization of  $n$

- When a new user gets a certificate containing  $e_{new}$ 
  - Recall:  $z = v^{\prod e_i} \bmod n$
  - Actually  $v$  never occurs anywhere...
    - so:  $v' = v^{1/e_{new}} \bmod n$  and  $x = z^{1/e_{new}} \bmod n$
  - Thus  $z$  needs not to be changed in case new member joins!
- Witnesses need to be recomputed upon revocation only!



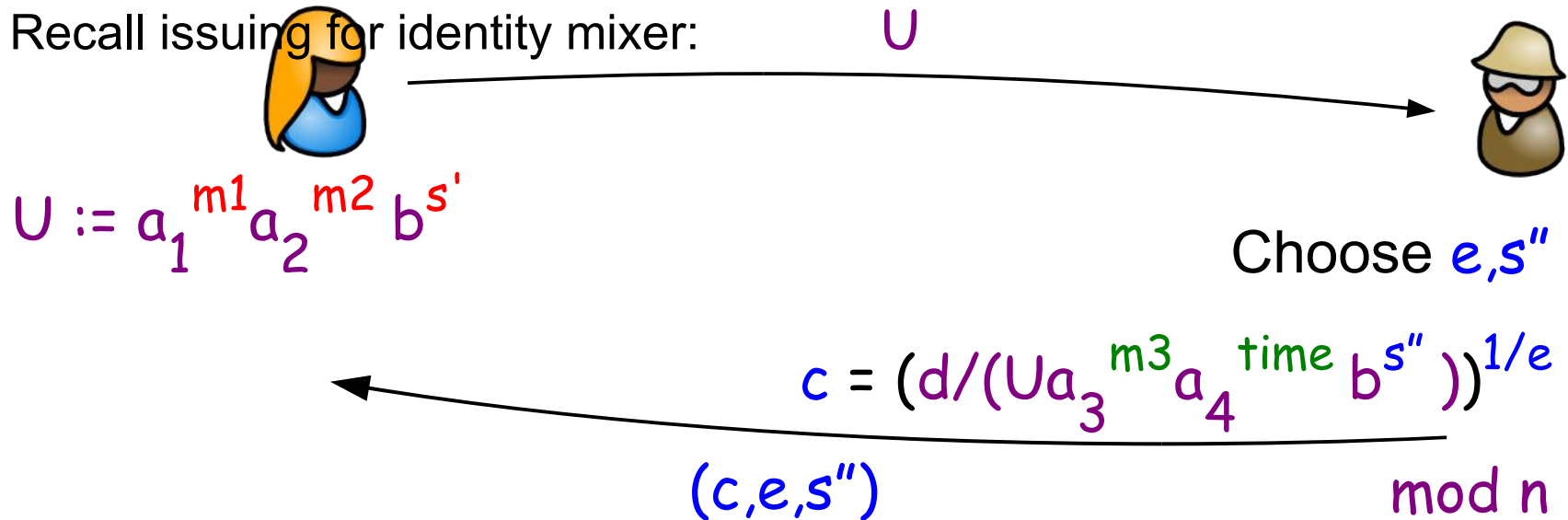
## Update of Credentials: encode validity time as attribute



Re-issue certificates

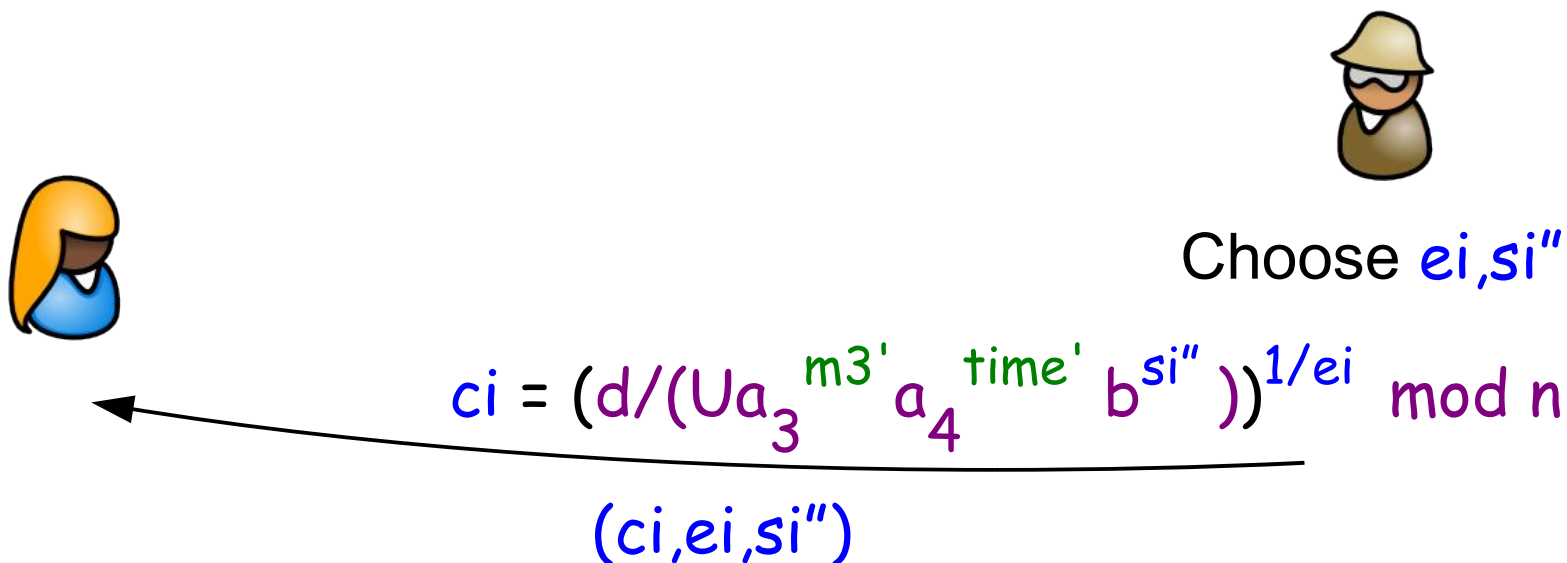
(off-line – interaction might be too expensive)

Recall issuing for identity mixer:



Re-issue certificates (off-line – interaction might be too expensive)

- Idea: just repeat last step for each new time **time'**:



- Update information  $(c_i, e_i, s_i'')$  can be pushed to user by many different means

## ■ Roadmap

- Explain possibilities to engineers, policy makers etc
- Usable prototypes
- Provide transparency
- Public infrastructure for privacy protection
- Laws with teeth (encourage investment in privacy)

## ■ Challenges

- Internet services get paid with personal data (inverse incentive)
- End users are not able to handle their data (user interfaces..)
- Security technology typically invisible and hard to sell

## ■ Towards a secure information society

- Society changes quickly and gets shaped by technology
- Consequences are hard to grasp (time will show...)
- We must inform and engage in a dialog

# Thank you!

- eMail: [identity@zurich.ibm.com](mailto:identity@zurich.ibm.com)
- Links:
  - [www.abc4trust.eu](http://www.abc4trust.eu)
  - [www.futureID.eu](http://www.futureID.eu)
  - [www.au2eu.eu](http://www.au2eu.eu)
  - [www.PrimeLife.eu](http://www.PrimeLife.eu)
  - [www.zurich.ibm.com/idemix](http://www.zurich.ibm.com/idemix)
  - [idemixdemo.zurich.ibm.com](http://idemixdemo.zurich.ibm.com)
- Code
  - [github.com/p2abcengine](https://github.com/p2abcengine) & [abc4trust.eu/idemix](http://abc4trust.eu/idemix)

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