Simulation Secure Multi-Input Quadratic Functional Encryption

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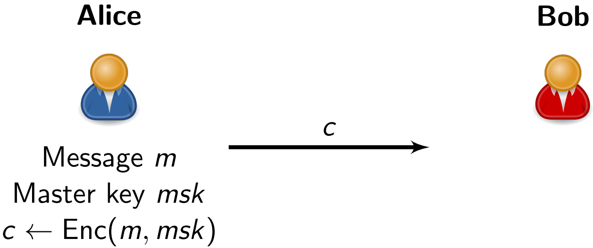
(Secret-key) Functional Encryption [BSW11, Boneh et al. TCC'11]



Alice

Bob

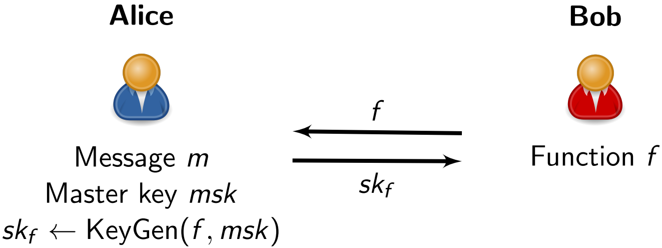
c

Message m

Master key msk

$c \leftarrow \mathrm{Enc}(m, msk)$

(Secret-key) Functional Encryption [BSW11, Boneh et al. TCC'11]



Alice

Message m
Master key msk
$sk_f \leftarrow \mathrm{KeyGen}(f, msk)$

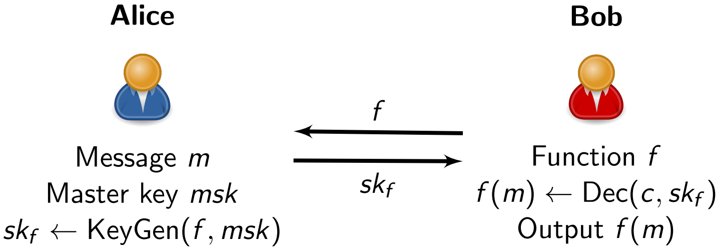
f

sk_f

Bob

Function f

(Secret-key) Functional Encryption [BSW11, Boneh et al. TCC'11]



Alice

Message m

Master key msk

$sk_f \leftarrow \text{KeyGen}(f, msk)$

f

sk_f

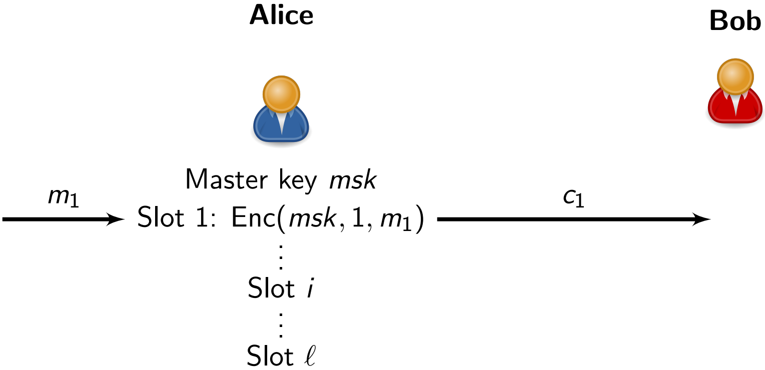
Bob

Function f

$f(m) \leftarrow \text{Dec}(c, sk_f)$

Output $f(m)$

(Secret-key) Multi-input Functional Encryption [GGG$^+$14, Goldwasser et al. EUROCRYPT'14]



Alice

Bob

m_1

Master key msk

Slot 1: $\mathsf{Enc}(msk, 1, m_1)$

c_1

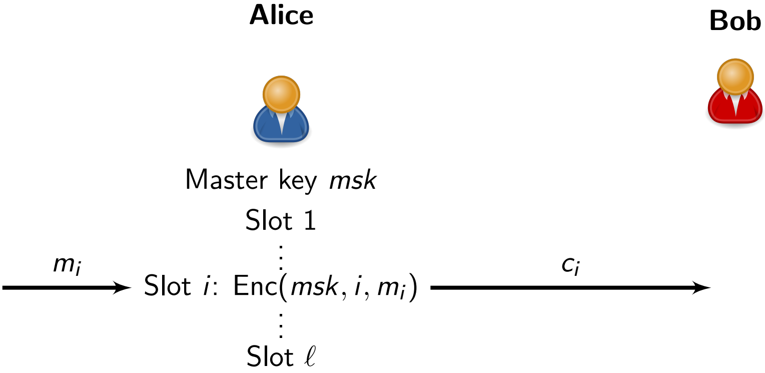
$\vdots$

Slot i

$\vdots$

Slot ℓ

(Secret-key) Multi-input Functional Encryption [GGG+14, Goldwasser et al. EUROCRYPT'14]



Alice

Bob

Master key *msk*

Slot 1

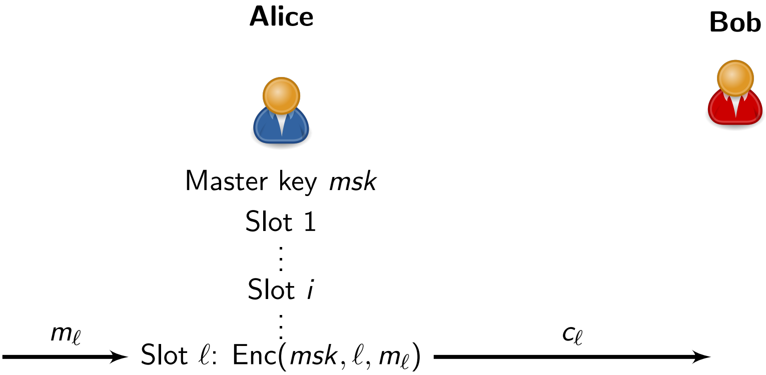
$\vdots$

$\xrightarrow{m_i}$ Slot i: $\mathsf{Enc}(msk, i, m_i)$ $\xrightarrow{c_i}$

$\vdots$

Slot ℓ

(Secret-key) Multi-input Functional Encryption [GGG+14, Goldwasser et al. EUROCRYPT'14]



Alice

Bob

Master key msk

Slot 1

$\vdots$

Slot i

$\vdots$

m_ℓ Slot ℓ: $\mathsf{Enc}(msk, \ell, m_\ell)$ c_ℓ

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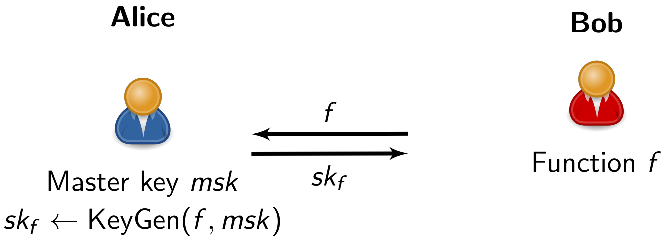


Alice

Master key msk

$sk_f \leftarrow \mathsf{KeyGen}(f, msk)$

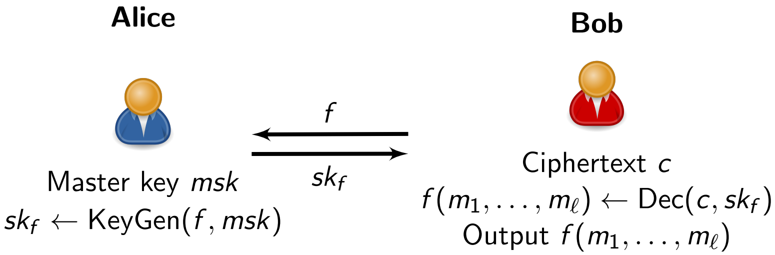
f

sk_f

Bob

Function f

(Secret-key) Multi-input Functional Encryption [GGG+14, Goldwasser et al. EUROCRYPT'14]



Alice

Master key msk

$sk_f \leftarrow \text{KeyGen}(f, msk)$

f

sk_f

Bob

Ciphertext c

$f(m_1, \ldots, m_\ell) \leftarrow \text{Dec}(c, sk_f)$

Output $f(m_1, \ldots, m_\ell)$

Applications of Multi-input Functional Encryption

- Searching over encrypted data [GGG$^+$14, Goldwasser et al. EUROCRYPT'14]
- Federated learning [XBZ$^+$19, Xu et al. AISec'19]
- Differential Privacy [AECLP24, Alborch Escobar et al. PETS'24]

Security of (Multi-Input) Functional Encryption

- Indistinguishability vs. simulation-based
  - Simulation-based stronger [AGVW13, Agrawal et al. CRYPTO'13] and more composable
  - Impossibility results for simulation-based ([BSW11, Boneh et al. TCC'11], ...)

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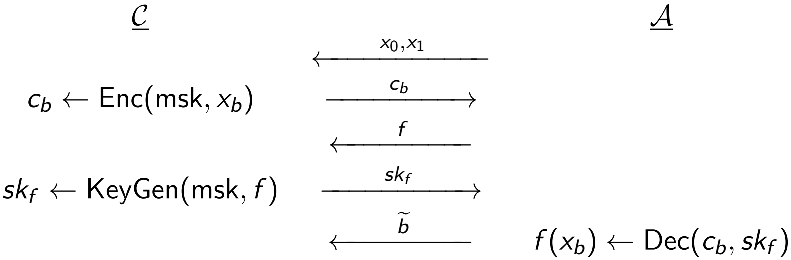
Experiment *b*:



$\underline{\mathcal{C}}$ $\underline{\mathcal{A}}$

$\xleftarrow{x_0, x_1}$

$c_b \leftarrow \mathsf{Enc}(\mathsf{msk}, x_b)$ $\xrightarrow{c_b}$

$\xleftarrow{f}$

$sk_f \leftarrow \mathsf{KeyGen}(\mathsf{msk}, f)$ $\xrightarrow{sk_f}$

$\xleftarrow{\widetilde{b}}$ $f(x_b) \leftarrow \mathsf{Dec}(c_b, sk_f)$

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Experiment *b*:

$\underline{\mathcal{C}}$ $\qquad\qquad\qquad$ $\underline{\mathcal{A}}$

$$\xleftarrow{\quad x_0, x_1 \quad}$$

$$c_b \leftarrow \mathrm{Enc}(\mathrm{msk}, x_b) \qquad \xrightarrow{\quad c_b \quad}$$

$$\xleftarrow{\quad f \quad}$$

$$sk_f \leftarrow \mathrm{KeyGen}(\mathrm{msk}, f) \qquad \xrightarrow{\quad sk_f \quad}$$

$$\xleftarrow{\quad \widetilde{b} \quad} \qquad f(x_b) \leftarrow \mathrm{Dec}(c_b, sk_f)$$

Needs $f(x_0) = f(x_1)$

Security of (Multi-Input) Functional Encryption

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```
$\mathsf{Exp}^{real}_{\mathcal{A}}(1^\lambda)$
$x \leftarrow \mathcal{A}(1^\lambda)$
$\mathsf{msk} \leftarrow \mathsf{SetUp}(1^\lambda)$
$c_x \leftarrow \mathsf{Enc}(\mathsf{msk}, x)$
$\gamma \leftarrow \mathcal{A}^{\mathsf{KeyGen}(\mathsf{msk}, f)}(c_x)$
```

```
$\mathsf{Exp}^{ideal}_{\mathcal{A},\mathsf{Sim}}(1^\lambda)$
$x \leftarrow \mathcal{A}(1^\lambda)$
$\widetilde{\mathsf{msk}} \leftarrow \mathsf{SetUpSim}(1^\lambda)$
$\widetilde{c} \leftarrow \mathsf{EncSim}(\widetilde{\mathsf{msk}})$
$\gamma \leftarrow \mathcal{A}^{\mathsf{KeyGenSim}(\widetilde{\mathsf{msk}}, f, f(x))}(\widetilde{c})$
```

Show real and ideal experiments are indistinguishable

Security of (Multi-Input) Functional Encryption

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- Selective vs. adaptive
  - Adaptive is stronger
  - Impossibility results for adaptive ([BSW11, Boneh et al. TCC'11], ...)

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- Selective vs. adaptive
  - Adaptive is stronger
  - Impossibility results for adaptive ([BSW11, Boneh et al. TCC'11], ...)

- Function-hiding functional encryption [SSW09, Shen et al. TCC'09]
  - Additional security property
  - Indistinguishability and simulation-based variants
  - Only in secret-key

State of the Art in MIFE

- Inner-product function: input $\boldsymbol{x}$ and function $\boldsymbol{y}$ output $\boldsymbol{x}^\top \boldsymbol{y}$ ($\sum \boldsymbol{x}_i^\top \boldsymbol{y}_i$ in multi-input)
  - Generic transformation from IPFE exists [ACF+18, Abdalla et al. CRYPTO'18].

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  - Generic transformation from IPFE exists [ACF+18, Abdalla et al. CRYPTO'18].
- Quadratic function: input $\boldsymbol{x}$ and function $\boldsymbol{F}$ output $\boldsymbol{x}^\top \boldsymbol{F} \boldsymbol{x}$ ($\sum \boldsymbol{x}_i^\top \boldsymbol{F}_{i,j} \boldsymbol{x}_{\boldsymbol{j}}$ in multi-input)

Table: State of the art. We consider ℓ inputs of size n or 1 input of size $n\ell$.

Proposal	Functionality	Simulation security	Ciphertext size
Naive	QFE	✓	$O(n^2\ell^2)$
[Gay20, Gay PKC'20]	QFE	✓	$O(n\ell)$
[AGT22, Agrawal et al. TCC'22]	MIQFE	✗	$O(n\ell)$

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[AGT22, Agrawal et al. TCC'22]	MIQFE	✗	$O(n\ell)$
Our construction	MIQFE	✓	$O(n\ell^2)$

Results I: Transformation from function-hiding IPFE to MIQFE

- Transformation from function-hiding IPFE to MIQFE keeping simulation security

```
$\underline{\mathbf{SetUp}^{\mathsf{MIQFE}}(1^\kappa)}$ :
$\mathcal{PG} \leftarrow \mathsf{PGGen}(1^\kappa)$
$\boldsymbol{u}_i \xleftarrow{\$} \mathbb{Z}_p^n,\ c_i \xleftarrow{\$} \mathbb{Z}_p,\ \boldsymbol{w}_{i,j} \xleftarrow{\$} \mathbb{Z}_p^{2n}\ \ i,j \in [\ell]$
$(param_{i,j}^{\mathsf{IPFE}}, \mathsf{msk}_{i,j}^{\mathsf{IPFE}}) \leftarrow \mathsf{SetUp}^{\mathsf{IPFE}}(1^\kappa, \mathcal{PG})$
$param^{\mathsf{MIQFE}} = \mathcal{PG}$
$\mathsf{msk}^{\mathsf{MIQFE}} = (\boldsymbol{u}_i, c_i, \boldsymbol{w}_{i,j}, \mathsf{msk}_{i,j}^{\mathsf{IPFE}})$
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$\underline{\mathbf{Enc}^{\mathsf{MIQFE}}(\mathsf{msk}^{\mathsf{MIQFE}}, i, \boldsymbol{x}_i)}$ :
$\boldsymbol{ct}_{\boldsymbol{x}_i} := \boldsymbol{x}_i + c_i \boldsymbol{u}_i$
$c_{i,j} \leftarrow \mathsf{Enc}^{\mathsf{IPFE}}\left(\mathsf{msk}_{i,j}^{\mathsf{IPFE}}, \boldsymbol{w}_{i,j} + c_j \begin{pmatrix} \boldsymbol{ct}_{\boldsymbol{x}_i} \\ \boldsymbol{x}_i \end{pmatrix}\right)$
$c_{\boldsymbol{x}_i} = (\boldsymbol{ct}_{\boldsymbol{x}_i}, c_{i,j})$
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$\underline{\mathbf{KeyGen}^{\mathsf{MIQFE}}(\mathsf{msk}^{\mathsf{MIQFE}}, \boldsymbol{F})}$ :
$sk_{i,j} \leftarrow \mathsf{KeyGen}^{\mathsf{IPFE}}\left(\mathsf{msk}_{i,j}^{\mathsf{IPFE}}, \begin{pmatrix} \boldsymbol{u}_j^\top \boldsymbol{F}_{j,i} \\ \boldsymbol{F}_{i,j} \boldsymbol{u}_j \end{pmatrix}\right)$
$zk_{\boldsymbol{F}} \leftarrow \sum_{i,j \in [\ell]} \boldsymbol{w}_{i,j}^\top \begin{pmatrix} \boldsymbol{u}_j^\top \boldsymbol{F}_{j,i} \\ \boldsymbol{F}_{i,j} \boldsymbol{u}_j \end{pmatrix}$
$sk_{\boldsymbol{F}} = (\boldsymbol{F}, sk_{i,j}, zk_{\boldsymbol{F}})$
```

```
$\underline{\mathbf{Dec}^{\mathsf{MIQFE}}(c_{\boldsymbol{x}_1}, \ldots, c_{\boldsymbol{x}_\ell}, sk_{\boldsymbol{F}})}$ :
$[d_{i,j}]_T \leftarrow \mathsf{IPFE.Dec}(\mathsf{IPFE}.c_{i,j}, \mathsf{IPFE}.sk_{i,j})$
$[v]_T := \left(\sum_{i,j \in [\ell]} [\boldsymbol{ct}_{\boldsymbol{x}_i}^\top \boldsymbol{F}_{i,j} \boldsymbol{ct}_{\boldsymbol{x}_j}]_T - [d_{i,j}]_T\right) + [zk_{\boldsymbol{F}}]_T$
$s \leftarrow \log([v]_T)$
```

Results I: Transformation from function-hiding IPFE to MIQFE

- Transformation from function-hiding IPFE to MIQFE keeping simulation security

One-time pad $\boldsymbol{ct}_{\boldsymbol{x_i}}$

Compute the quadratic function over $\boldsymbol{ct}_{\boldsymbol{x_i}}$

$\underline{\mathbf{Enc}^{\mathsf{MIQFE}}(\mathsf{msk}^{\mathsf{MIQFE}}, i, \boldsymbol{x}_i)}$:

$\boldsymbol{ct}_{\boldsymbol{x}_i} := \boldsymbol{x}_i + c_i \boldsymbol{u}_i$

$c_{i,j} \leftarrow \mathsf{Enc}^{\mathsf{IPFE}}\left(\mathsf{msk}^{\mathsf{IPFE}}_{i,j}, \boldsymbol{w}_{i,j} + c_j \begin{pmatrix} \boldsymbol{ct}_{\boldsymbol{x}_i} \\ \boldsymbol{x}_i \end{pmatrix}\right)$

$c_{\boldsymbol{x}_i} = (\boldsymbol{ct}_{\boldsymbol{x}_i}, c_{i,j})$

Extra noise terms:
$c_i \boldsymbol{u}_i^\top \boldsymbol{F}_{i,j} \boldsymbol{x}_j + \boldsymbol{x}_i^\top \boldsymbol{F}_{i,j} c_j \boldsymbol{u}_j + c_i \boldsymbol{u}_i^\top \boldsymbol{F}_{i,j} c_j \boldsymbol{u}_j$

$\underline{\mathbf{Dec}^{\mathsf{MIQFE}}(c_{\boldsymbol{x}_1}, \ldots, c_{\boldsymbol{x}_\ell}, sk_{\boldsymbol{F}})}$:

$[d_{i,j}]_T \leftarrow \mathsf{IPFE.Dec}(\mathsf{IPFE}.c_{i,j}, \mathsf{IPFE}.sk_{i,j})$

$[v]_T := \left(\sum_{i,j\in[\ell]} [\boldsymbol{ct}_{\boldsymbol{x}_i}^\top \boldsymbol{F}_{i,j} \boldsymbol{ct}_{\boldsymbol{x}_j}]_T - [d_{i,j}]_T\right) + [zk_{\boldsymbol{F}}]_T$

$s \leftarrow \log([v]_T)$

Results I: Transformation from function-hiding IPFE to MIQFE

- Transformation from function-hiding IPFE to MIQFE keeping simulation security

Use IPFE to compute extra terms

“Interweave” terms from $\boldsymbol{F}_{i,j}$ and $\boldsymbol{F}_{j,i}$, in $d_{i,j}$:

Compute $c_j \boldsymbol{u}_j^\top \boldsymbol{F}_{j,i} \boldsymbol{x}_i + \boldsymbol{x}_j^\top \boldsymbol{F}_{j,i} c_i \boldsymbol{u}_i + c_j \boldsymbol{u}_j^\top \boldsymbol{F}_{j,i} c_i \boldsymbol{u}_i$ for j, i

Compute $c_i \boldsymbol{u}_i^\top \boldsymbol{F}_{i,j} \boldsymbol{x}_j + \boldsymbol{x}_i^\top \boldsymbol{F}_{i,j} c_j \boldsymbol{u}_j + c_i \boldsymbol{u}_i^\top \boldsymbol{F}_{i,j} c_j \boldsymbol{u}_j$ for i, j

$\underline{\mathbf{Enc}^{\mathsf{MIQFE}}(\mathsf{msk}^{\mathsf{MIQFE}}, i, \boldsymbol{x}_i)}$:

$$\boldsymbol{ct}_{\boldsymbol{x}_i} := \boldsymbol{x}_i + c_i \boldsymbol{u}_i$$
$$c_{i,j} \leftarrow \mathsf{Enc}^{\mathsf{IPFE}}\left(\mathsf{msk}^{\mathsf{IPFE}}_{i,j}, \boldsymbol{w}_{i,j} + c_j \begin{pmatrix} \boldsymbol{ct}_{\boldsymbol{x}_i} \\ \boldsymbol{x}_i \end{pmatrix}\right)$$
$$c_{\boldsymbol{x}_i} = (\boldsymbol{ct}_{\boldsymbol{x}_i}, c_{i,j})$$

$\underline{\mathbf{KeyGen}^{\mathsf{MIQFE}}(\mathsf{msk}^{\mathsf{MIQFE}}, \boldsymbol{F})}$:

$$sk_{i,j} \leftarrow \mathsf{KeyGen}^{\mathsf{IPFE}}\left(\mathsf{msk}^{\mathsf{IPFE}}_{i,j}, \begin{pmatrix} \boldsymbol{u}_j^\top \boldsymbol{F}_{j,i} \\ \boldsymbol{F}_{i,j} \boldsymbol{u}_j \end{pmatrix}\right)$$
$$zk_{\boldsymbol{F}} \leftarrow \sum_{i,j \in [\ell]} \boldsymbol{w}_{i,j}^\top \begin{pmatrix} \boldsymbol{u}_j^\top \boldsymbol{F}_{j,i} \\ \boldsymbol{F}_{i,j} \boldsymbol{u}_j \end{pmatrix}$$
$$sk_{\boldsymbol{F}} = (\boldsymbol{F}, sk_{i,j}, zk_{\boldsymbol{F}})$$

$\underline{\mathbf{Dec}^{\mathsf{MIQFE}}(c_{\boldsymbol{x}_1}, \ldots, c_{\boldsymbol{x}_\ell}, sk_{\boldsymbol{F}})}$:

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$$s \leftarrow \log([v]_T)$$

Results I: Transformation from function-hiding IPFE to MIQFE

- Transformation from function-hiding IPFE to MIQFE keeping simulation security

One-time pad $\boldsymbol{w}$ for IPFE input

To ensure output can only be recovered with all inputs

$\underline{\textbf{KeyGen}^{\text{MIQFE}}(\text{msk}^{\text{MIQFE}}, \boldsymbol{F})}$:

$$sk_{i,j} \leftarrow \text{KeyGen}^{\text{IPFE}}\left(\text{msk}^{\text{IPFE}}_{i,j}, \begin{pmatrix} \boldsymbol{u}_j^\top \boldsymbol{F}_{j,i} \\ \boldsymbol{F}_{i,j}\boldsymbol{u}_j \end{pmatrix}\right)$$

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$$sk_{\boldsymbol{F}} = (\boldsymbol{F}, sk_{i,j}, zk_{\boldsymbol{F}})$$

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$$c_{\boldsymbol{x}_i} = (\boldsymbol{ct}_{\boldsymbol{x}_i}, c_{i,j})$$

$\underline{\textbf{Dec}^{\text{MIQFE}}(c_{\boldsymbol{x}_1}, \ldots, c_{\boldsymbol{x}_\ell}, sk_{\boldsymbol{F}})}$:

$$[d_{i,j}]_T \leftarrow \text{IPFE.Dec}(\text{IPFE}.c_{i,j}, \text{IPFE}.sk_{i,j})$$

$$[v]_T := \left(\sum_{i,j\in[\ell]} [\boldsymbol{ct}_{\boldsymbol{x}_i}^\top \boldsymbol{F}_{i,j} \boldsymbol{ct}_{\boldsymbol{x}_j}]_T - [d_{i,j}]_T\right) + [zk_{\boldsymbol{F}}]_T$$

$$s \leftarrow \log([v]_T)$$

Results I: Transformation from function-hiding IPFE to MIQFE

- Transformation from function-hiding IPFE to MIQFE keeping simulation security

Theorem

The MIQFE *scheme is one selective multi-input simulation secure, if the underlying inner-product functional encryption scheme is one selective function-hiding simulation secure. In other words, for any* PPT *adversary $\mathcal{A}$ there exist* PPT *adversaries $\mathcal{B}$ such that*

$$\mathsf{Adv}^{\mathsf{MI-SIM}}_{\mathsf{MIQFE}}(\mathcal{A}) \leq \ell^2 \cdot \mathsf{Adv}^{\mathsf{FH\text{-}SIM}}_{\mathsf{IPFE}}(\mathcal{B}) + \frac{\ell}{p}.$$

- Proof intuition: First simulate $\boldsymbol{ct}_{\boldsymbol{x}_i}$ with uniformly at random and modify the rest accordingly. Then swap for the ℓ^2 function-hiding IPFE simulators. Finally use that $\boldsymbol{w}_{i,j}$ are uniformly at random to simulate $d_{i,j}$.

Results II: function-hiding IPFE

- We need simulation secure function-hiding IPFE

Table: State of the art.

Proposal	Functionality	Simulation security	Model
[Lin17, Lin CRYPTO'17]	FH-IPFE	✗	Standard
[KLM$^+$18, Kim et al. SCN'18]	FH-IPFE	✓	GGM

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[KLM$^+$18, Kim et al. SCN'18]	FH-IPFE	✓	GGM
Our construction	FH-IPFE	✓	Standard

Results II: function-hiding IPFE

- Pairing-based from nesting twice an IPFE scheme
  - [ABCP15, Abdalla et al. PKC'15], for msk $= \boldsymbol{u}$ then

$$\underline{\mathbf{KeyGen}^{\mathsf{IPFE}}(\mathsf{msk}^{\mathsf{IPFE}}, \boldsymbol{y})}:$$

$$sk_{\boldsymbol{y}} = \begin{pmatrix} -\boldsymbol{u}^\top \boldsymbol{y} \\ \boldsymbol{y} \end{pmatrix}$$

Results II: function-hiding IPFE

- Pairing-based from nesting twice an IPFE scheme
  - [ABCP15, Abdalla et al. PKC'15], for $\mathsf{msk} = \boldsymbol{u}$ then

$$\underline{\textbf{KeyGen}^{\mathsf{IPFE}}(\mathsf{msk}^{\mathsf{IPFE}}, \boldsymbol{y})}: \\ sk_{\boldsymbol{y}} = \begin{pmatrix} -\boldsymbol{u}^\top \boldsymbol{y} \\ \boldsymbol{y} \end{pmatrix}$$

- To solve this

$$\mathsf{KeyGen}^{\mathsf{IPFE}}(\boldsymbol{y}) = \mathsf{KeyGen}^{\mathsf{out}}(\mathsf{Enc}^{\mathsf{in}}(\boldsymbol{y})) \mid \mathsf{Enc}^{\mathsf{IPFE}}(\boldsymbol{x}) = \mathsf{Enc}^{\mathsf{out}}(\mathsf{KeyGen}^{\mathsf{in}}(\boldsymbol{x}))$$

Results II: function-hiding IPFE

Pairing-friendly groups $\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T$

$$e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$$
$$e([a]_1, [b]_2) \to [a \cdot b]_T$$

- Pairing-based from nesting twice an IPFE scheme

$$\mathsf{KeyGen}^{\mathsf{IPFE}}(\boldsymbol{y}) = \mathsf{KeyGen}^{\mathsf{out}}(\mathsf{Enc}^{\mathsf{in}}(\boldsymbol{y})) \mid \mathsf{Enc}^{\mathsf{IPFE}}(\boldsymbol{x}) = \mathsf{Enc}^{\mathsf{out}}(\mathsf{KeyGen}^{\mathsf{in}}(\boldsymbol{x}))$$

$\underline{\mathbf{SetUp}^{\mathsf{IPFE}}(1^\kappa, \mathcal{PG})}$:

$\boldsymbol{u} \overset{\$}{\leftarrow} \mathbb{Z}_p^{n+1}$, $\boldsymbol{v} \overset{\$}{\leftarrow} \mathbb{Z}_p^n$

$param^{\mathsf{IPFE}} = \mathcal{PG}$

$\mathsf{msk}^{\mathsf{IPFE}} = (\boldsymbol{u}, \boldsymbol{v})$

$\underline{\mathbf{KeyGen}^{\mathsf{IPFE}}(\mathsf{msk}^{\mathsf{IPFE}}, \boldsymbol{y})}$:

$t \overset{\$}{\leftarrow} \mathbb{Z}_p$

$$sk_1 := \left[-\boldsymbol{u}^\top \begin{pmatrix} t \\ \boldsymbol{y} + t \cdot \boldsymbol{v} \end{pmatrix}\right]_2, \; sk_2 := \left[\begin{pmatrix} t \\ \boldsymbol{y} + t \cdot \boldsymbol{v} \end{pmatrix}\right]_2$$

$sk_{\boldsymbol{y}} = (sk_1, sk_2)$

$\underline{\mathbf{Enc}^{\mathsf{IPFE}}(\mathsf{msk}^{\mathsf{IPFE}}, \boldsymbol{x})}$:

$c \overset{\$}{\leftarrow} \mathbb{Z}_p$

$$ct_1 := [c]_1, \; ct_2 := \left[\begin{pmatrix} -\boldsymbol{v}^\top \boldsymbol{x} \\ \boldsymbol{x} \end{pmatrix} + c \cdot \boldsymbol{u}\right]_1$$

$c_{\boldsymbol{x}} = (ct_1, ct_2)$

$\underline{\mathbf{Dec}^{\mathsf{IPFE}}(c_{\boldsymbol{x}}, sk_{\boldsymbol{y}})}$:

$[v]_T := e(ct_1, sk_1) + e(ct_2, sk_2)$

$s \leftarrow \log([v]_T)$

Results II: function-hiding IPFE

Pairing-friendly groups $\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T$

$$e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$$
$$e([a]_1, [b]_2) \to [a \cdot b]_T$$

- Pairing-based from nesting twice an IPFE scheme

$$\mathsf{KeyGen}^{\mathsf{IPFE}}(\boldsymbol{y}) = \mathsf{KeyGen}^{\mathsf{out}}(\mathsf{Enc}^{\mathsf{in}}(\boldsymbol{y})) \mid \mathsf{Enc}^{\mathsf{IPFE}}(\boldsymbol{x}) = \mathsf{Enc}^{\mathsf{out}}(\mathsf{KeyGen}^{\mathsf{in}}(\boldsymbol{x}))$$

$\underline{\textbf{SetUp}^{\mathsf{IPFE}}(1^\kappa, \mathcal{PG})}$:

$\boldsymbol{u} \xleftarrow{\$} \mathbb{Z}_p^{n+1}$, $\boldsymbol{v} \xleftarrow{\$} \mathbb{Z}_p^{n}$

$param^{\mathsf{IPFE}} = \mathcal{PG}$

$\mathsf{msk}^{\mathsf{IPFE}} = (\boldsymbol{u}, \boldsymbol{v})$

$\underline{\textbf{KeyGen}^{\mathsf{IPFE}}(\mathsf{msk}^{\mathsf{IPFE}}, \boldsymbol{y})}$:

$t \xleftarrow{\$} \mathbb{Z}_p$

$$sk_1 := \left[-\boldsymbol{u}^\top \begin{pmatrix} t \\ \boldsymbol{y} + t \cdot \boldsymbol{v} \end{pmatrix} \right]_2, \; sk_2 := \left[\begin{pmatrix} t \\ \boldsymbol{y} + t \cdot \boldsymbol{v} \end{pmatrix} \right]_2$$

$sk_{\boldsymbol{y}} = (sk_1, sk_2)$

$\underline{\textbf{Enc}^{\mathsf{IPFE}}(\mathsf{msk}^{\mathsf{IPFE}}, \boldsymbol{x})}$:

$c \xleftarrow{\$} \mathbb{Z}_p$

$$ct_1 := [c]_1, \; ct_2 := \left[\begin{pmatrix} -\boldsymbol{v}^\top \boldsymbol{x} \\ \boldsymbol{x} \end{pmatrix} + c \cdot \boldsymbol{u} \right]_1$$

$c_{\boldsymbol{x}} = (ct_1, ct_2)$

$\underline{\textbf{Dec}^{\mathsf{IPFE}}(c_{\boldsymbol{x}}, sk_{\boldsymbol{y}})}$:

$[v]_T := e(ct_1, sk_1) + e(ct_2, sk_2)$

$s \leftarrow \log([v]_T)$

Results II: function-hiding IPFE

- Pairing-based from nesting twice an IPFE scheme

Theorem

The IPFE *scheme is one selective function-hiding simulation secure, if the* DDH *assumption holds in group* $\mathbb{G}_2$*. In other words, for any* PPT *adversary* $\mathcal{A}$ *there exists a* PPT *adversary* $\mathcal{B}$ *such that*

$$\mathsf{Adv}^{\mathsf{FH\text{-}SIM}}_{\mathsf{IPFE}}(\mathcal{A}) \leq 2Q_{sk} \cdot \mathsf{Adv}^{\mathsf{DDH}}_{\mathbb{G}_2}(\mathcal{B}) + \frac{1}{p} + \frac{2Q_{sk}}{p-1}.$$

where Q_{sk} *denotes the number of queries performed to* KeyGen*.*

- Proof intuition: First simulate ct_2 with uniformly at random. Then use the n-fold DDH assumption for each functional key query to simulate the functional keys.

Efficiency Considerations and Open Problems

Table: Efficiency estimates for our MIQFE and IPFE constructions.

	Secret key	Ciphertext (per input)	Functional key
Generic MIQFE	$\ell^2 \cdot \mathsf{IPFE}^{2n}_{\mathsf{msk}} + \ell(1+n)\lvert p\rvert + \ell^2 2n\lvert p\rvert$	$\ell \cdot \mathsf{IPFE}^{2n}_{c_x} + n\lvert p\rvert$	$\ell^2 \cdot \mathsf{IPFE}^{2n}_{sk_y} + \lvert p\rvert$
FH-IPFE	$(2n+1)\lvert p\rvert$	$(n+2)\lvert\mathbb{G}_1\rvert$	$(n+2)\lvert\mathbb{G}_2\rvert$
Concrete MIQFE	$\ell^2(4n+1)\lvert p\rvert + \ell(1+n)\lvert p\rvert + \ell^2 2n\lvert p\rvert$	$\ell \cdot (2n+2)\lvert\mathbb{G}_1\rvert + n\lvert p\rvert$	$\ell^2 \cdot (2n+2)\lvert\mathbb{G}_2\rvert + \lvert p\rvert$

Efficiency Considerations and Open Problems

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Generic MIQFE	$\ell^2 \cdot \mathsf{IPFE}^{2n}_{\mathsf{msk}} + \ell(1+n)\lvert p\rvert + \ell^2 2n\lvert p\rvert$	$\ell \cdot \mathsf{IPFE}^{2n}_{c_x} + n\lvert p\rvert$	$\ell^2 \cdot \mathsf{IPFE}^{2n}_{sk_y} + \lvert p\rvert$
FH-IPFE	$(2n+1)\lvert p\rvert$	$(n+2)\lvert\mathbb{G}_1\rvert$	$(n+2)\lvert\mathbb{G}_2\rvert$
Concrete MIQFE	$\ell^2(4n+1)\lvert p\rvert + \ell(1+n)\lvert p\rvert + \ell^2 2n\lvert p\rvert$	$\ell \cdot (2n+2)\lvert\mathbb{G}_1\rvert + n\lvert p\rvert$	$\ell^2 \cdot (2n+2)\lvert\mathbb{G}_2\rvert + \lvert p\rvert$

Open problems:

- Improving ciphertext size to $O(n\ell)$
- Transformation directly from QFE

Thank you for your attention

Questions?

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