Minimize the Randomness in Rasta-Like Designs: How Far Can We Go?

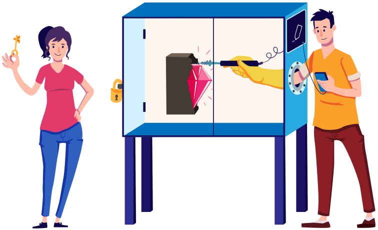
Application to PASTA

Lorenzo Grassi, Fukang Liu, Christian Rechberger,
Fabian Schmid, Roman Walch, and Qingju Wang

28.08.2024

Homomorphic Encryption

- HE scheme $\mathcal{E}$ is set of functions:
  - $\texttt{Setup}, \texttt{Enc}, \texttt{Dec}, \texttt{KeyGen}, \texttt{Eval}$
- Outsourcing computation on encrypted data
- $\mathcal{E}$ introduces noise and Ciphertext Expansion
  - Depending on $\mathcal{E}.\texttt{Eval}$
- Applications are faced with complex trade-offs:
  - Plaintext precision
  - Evaluation complexity
  - Security
  - Performance (Computation, Communication)



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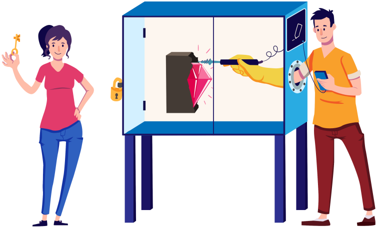

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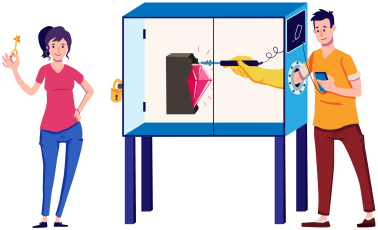

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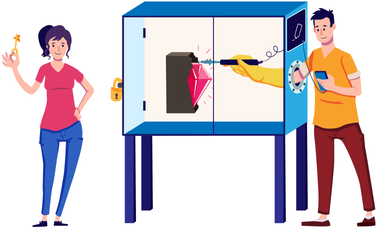

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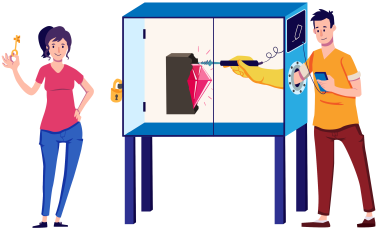

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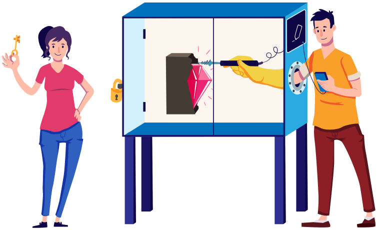

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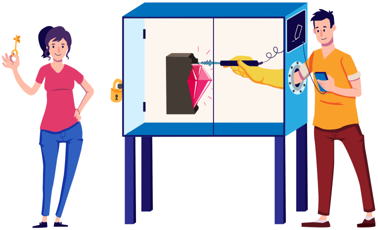

Parameters and Ciphertext Expansion

- Consider the polynomial ring $R = \mathbb{Z}[X]/(X^n + 1)$
  - Ciphertext and Plaintext spaces R_q and R_t, where $q >> t$
- With perfect parallelization, the expansion factor is at least $2 \cdot \lceil \frac{q}{t} \rceil$
  - Can be $\geq$ 100x for complex use cases
  - Much worse without parallelization
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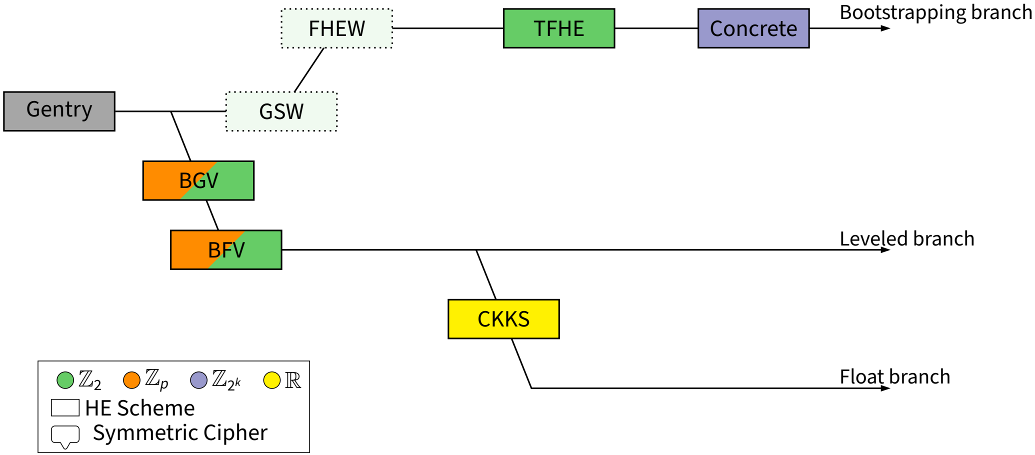
Ciphers for HHE

A Zoo of Ciphers for HHE



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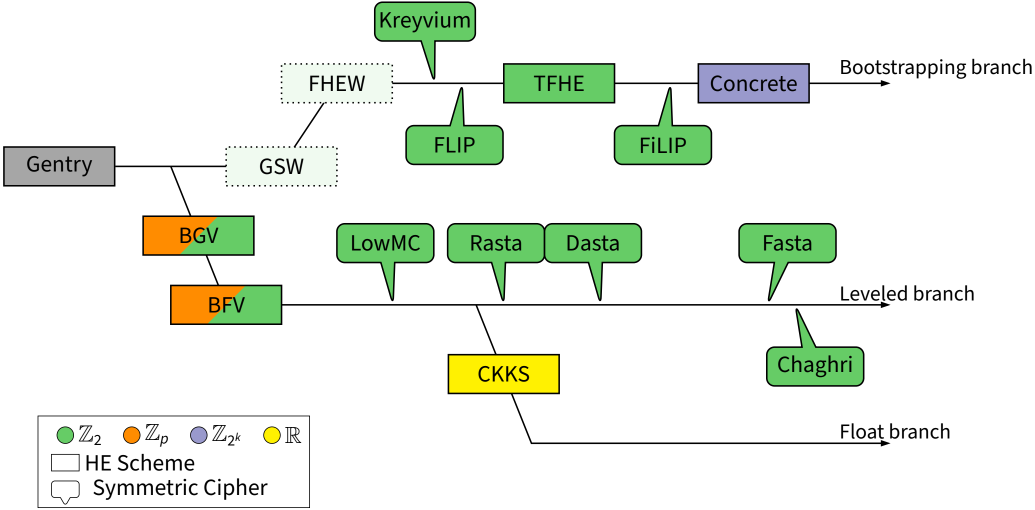

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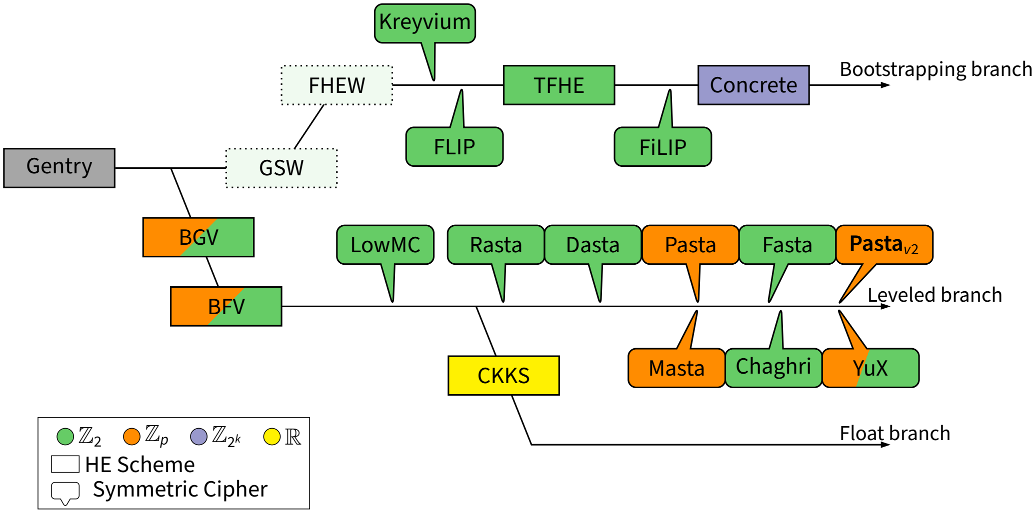

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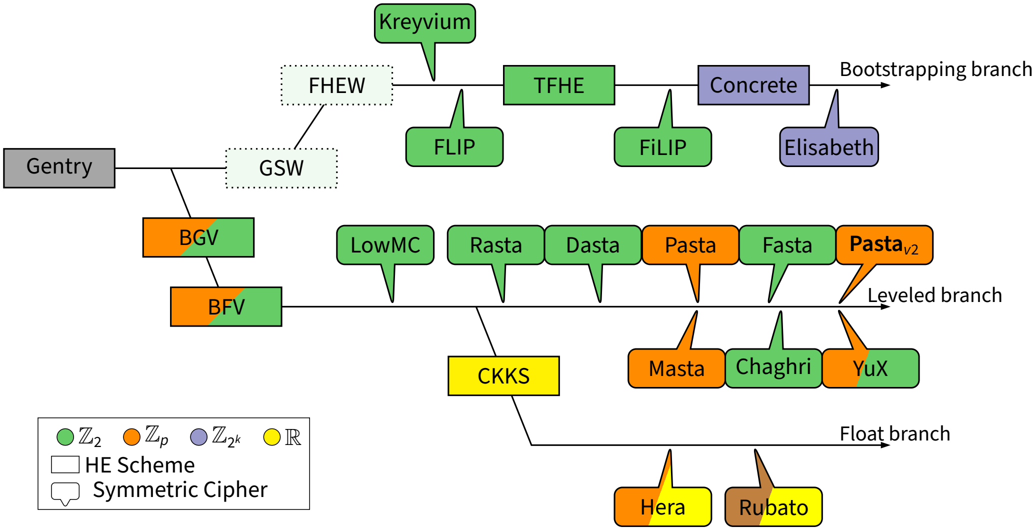
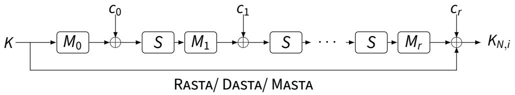


Design of PASTA$_{v2}$

Background: Design of RASTA-like Ciphers

Randomized Stream-Ciphers

- RASTA:
  - Random invertible matrices
  - Random round constants
- DASTA:
  - Improved matrix generation
- MASTA:
  - RASTA strategy applied to $\mathbb{F}_p$



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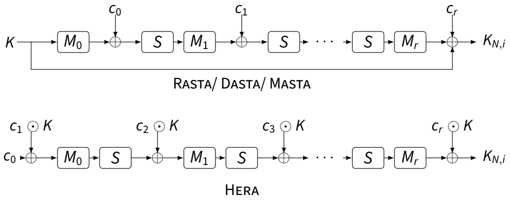
Randomized Stream-Ciphers

- HERA:
  - Fixed matrices
  - Randomized round keys
  - Small statesize



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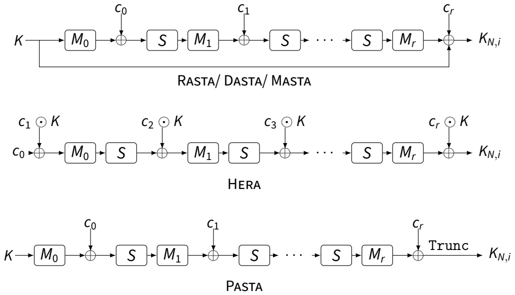
Randomized Stream-Ciphers

- PASTA:
  - Matrices with high branch number
  - Truncation of output
  - Geared towards HE evaluation



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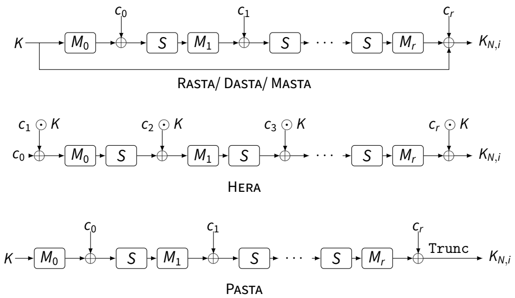
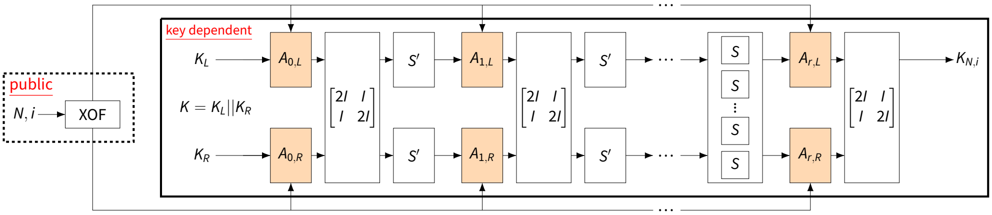


Randomization dominates encryption cost

The PASTA Design Strategy – Linear Layer



Linear Layer:

$$\begin{bmatrix} \vec{y}_L \\ \vec{y}_R \end{bmatrix} = \begin{bmatrix} 2 \cdot I & I \\ I & 2 \cdot I \end{bmatrix} \times \begin{bmatrix} M_{j,L,N,i}(\vec{x}_L) + c_{j,L,N,i} \\ M_{j,R,N,i}(\vec{x}_R) + c_{j,R,N,i} \end{bmatrix}$$

- Different random matrices and constants in each round.

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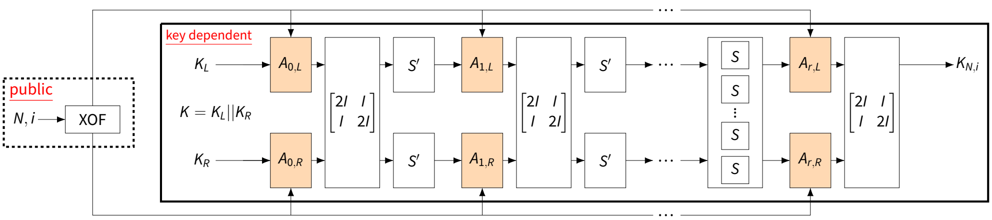

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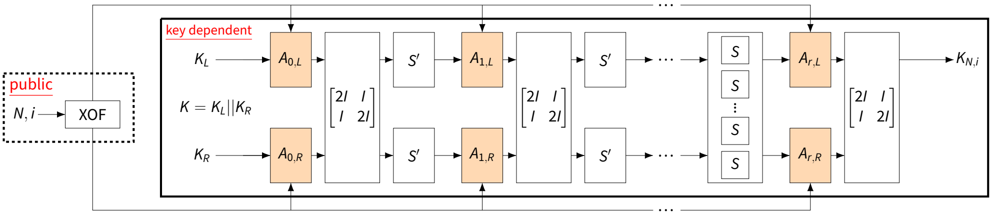

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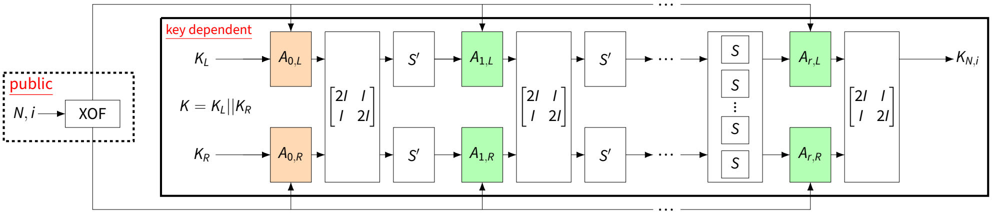
The Birth of PASTA$_{v2}$



The PASTA$_{v2}$ design

- Replace some random with fixed affine layers

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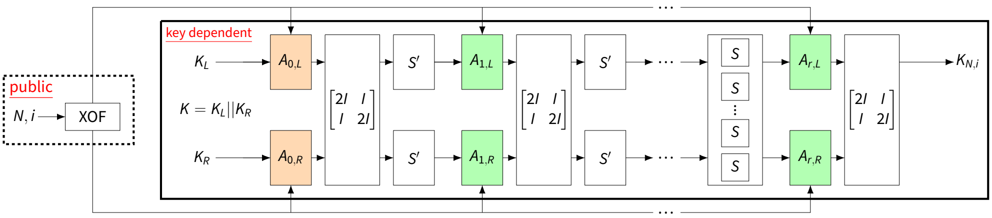

The PASTA$_{v2}$ design

	PASTA	PASTA$_{v2}$
One-time Setup	-	244 052
Affine Gen	23 550	6 200
Setup/Block	246 995	13 099

Table: Setup generation cost in CPU cycles

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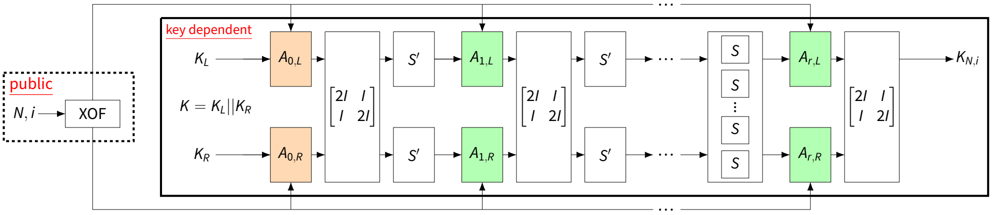

The PASTA$_{v2}$ design

$$\text{PASTA}_{v2}\text{-}\pi(x, N, i) = A_r \circ S \circ A_{r-1} \circ S' \circ \cdots \circ A_1 \circ S' \circ A_{0,N,i}(x)$$

Randomized Linear Layer

- Define fixed $M_{f,L}$ and $M_{f,R}$ as in PASTA
- During encryption, sample $2t$ random elements $(\beta_1, \ldots, \beta_{2t})$ and generate:

$$M_{0,L,N,i} = M_{f,L} \times \text{diag}(b_1, \ldots, \beta_t)$$
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- Only $4t$ random elements per encryption
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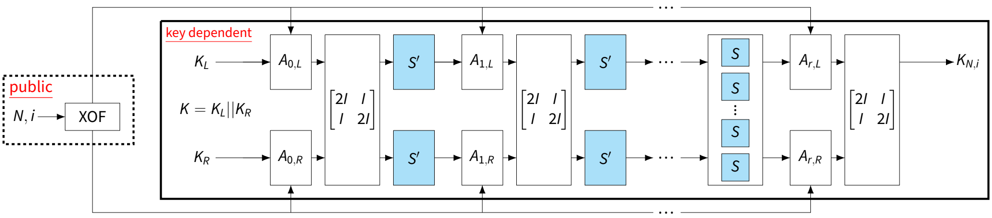
We provide a proof that the branch number of A_j is t + 2

The Non-Linear Layers



- Feistel-like S-box:
  - Low-degree ⇒ low depth

$$[S'(\vec{x})]_i = \begin{cases} x_0 & \text{if } i = 0 \\ x_i + (x_{i-1})^2 & \text{else} \end{cases}$$

- Cube S-box:
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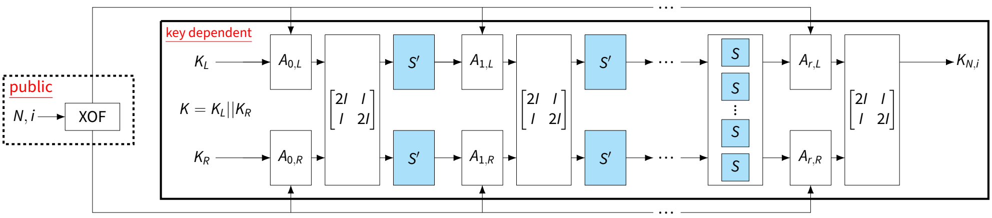

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Security Analysis

- Randomization provides resistance against:
  - Differential, truncated differential, and impossible differential attacks
  - Cube attacks and higher order differentials
- Linear Cryptanalysis breaking PASTA_{v2} reduced to LWE
  - High minimum of active non-linear operations
- Algebraic Attacks set up independent variables for all monomials
  - Experiments showed a high number of monomials
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PASTA$_{v2}$ Instances

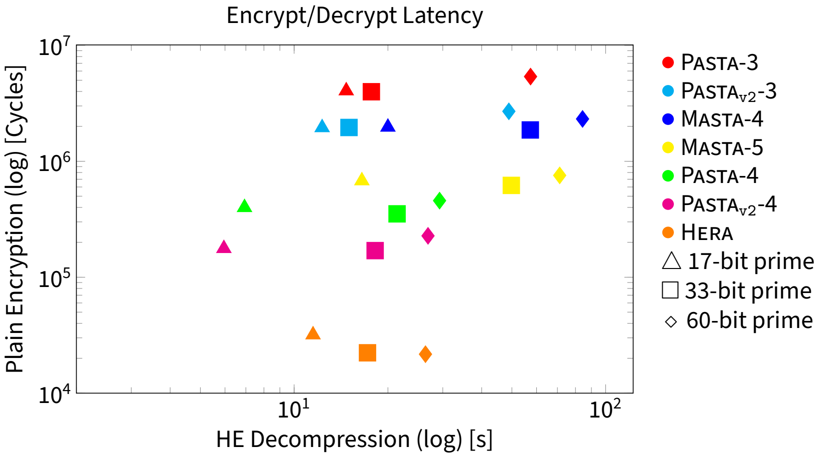
- We specify instances with the same security level as PASTA

Instance	r	# Key Words	# Plain/Cipher Words	# random words
PASTA$_{v2}$-3	3	256	128	512
PASTA$_{v2}$-4	4	64	32	128
PASTA-3	3	256	128	2048
PASTA-4	4	64	32	640

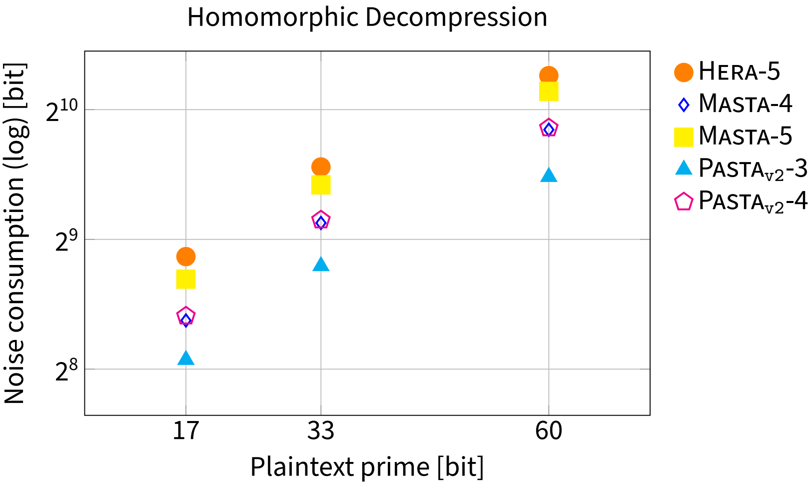
Table: 128 bit security instances of PASTA$_{v2}$ and PASTA

Benchmarks

Overall Performance



Noise development



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- We provide open-source implementation
  - Integration with HHE benchmarking framework[1]
  - HE Decompression implementation in SEAL and HElib
  - C++ plaintext implementation for encryption
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  - Faster Encryption and slightly faster Homomorphic decompression
  - Provably high branch number in fixed linear layers
  - Same security level for a fraction of required random words
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