An attack on ML-DSA using an implicit hint

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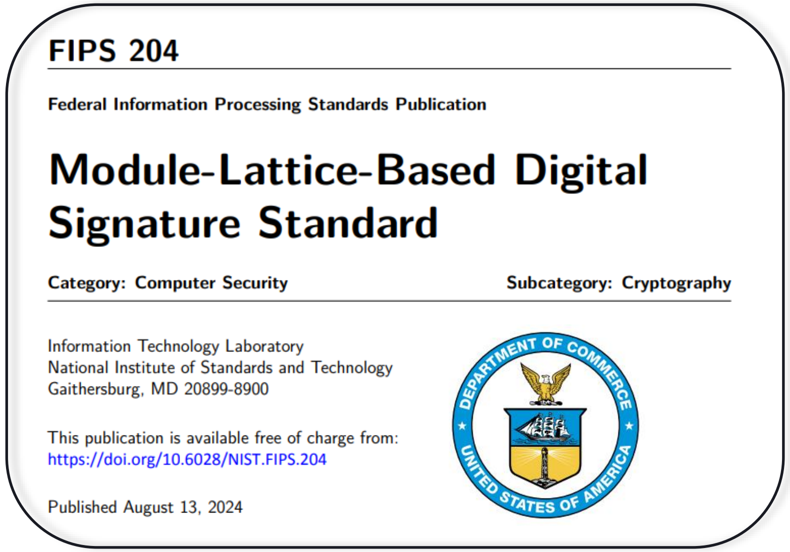
Context

Dilithium is a signature algorithm recently standardized by NIST under the name ML-DSA.

ML-DSA is recommended for computing quantum-secure signatures in most use cases.



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Module-Lattice-Based Digital Signature Standard

Category: Computer Security Subcategory: Cryptography

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Gaithersburg, MD 20899-8900

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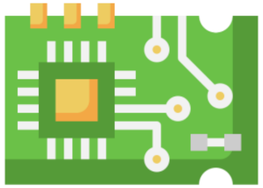
$m \rightarrow$  $\rightarrow \sigma_m$

it is necessary to investigate the security of embedded implementations. The security of ML-DSA against Side-Channel Attacks (SCA) and Fault Attacks (FA) thus needs to be carefully assessed.

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An overview of ML-DSA

ML-DSA uses two rings:

$$\mathcal{R} = \mathbb{Z}[x]/(x^n + 1) \qquad \mathcal{R}_q = \mathbb{Z}_q[x]/(x^n + 1)$$

with: $n = 256$ and $q = 8380417$.

Algorithm **KeyGen**

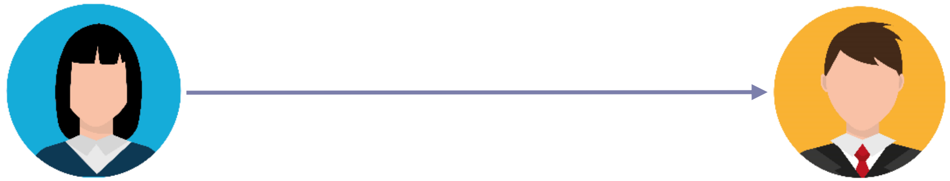
Ensure: (pk, sk)

$\mathbf{A} \leftarrow \mathcal{R}_q^{k \times l}$

$(\mathbf{s}_1, \mathbf{s}_2) \leftarrow S_\eta^l \times S_\eta^k$

$\mathbf{t} := \mathbf{A}\,\mathbf{s}_1 + \mathbf{s}_2$

return $pk = (\mathbf{A}, \mathbf{t}),\ sk = (\mathbf{A}, \mathbf{t}, \mathbf{s}_1, \mathbf{s}_2)$



(A, t, s_1, s_2) (A, t)

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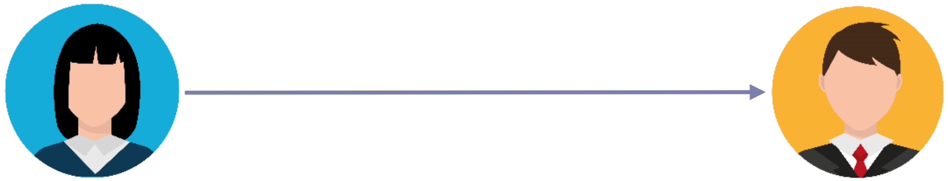

(A, t, s_1, s_2) (A, t)

α an even integer which divides $q - 1$ and:

$$r = r_1\alpha + r_0 \; with \; r_0 = r \bmod^{\pm}(\alpha) \; and \; r_1 = \frac{r - r_0}{\alpha}$$

Possible values of r_0: $\left\{-\frac{\alpha}{2} + 1, \dots, 0, \dots, \frac{\alpha}{2}\right\}$

Possible values of $r_1\alpha$: $\{0, \alpha, 2\alpha, \dots, q - 1\}$

One note:

$HighBits_q(r, \alpha) = r_1$ and $LowBits_q(r, \alpha) = r_0$

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with: $n = 256$ and $q = 8380417$

$$r = HighBits_q(r, \alpha) \times \alpha + LowBits_q(r, \alpha)$$

Algorithm KeyGen

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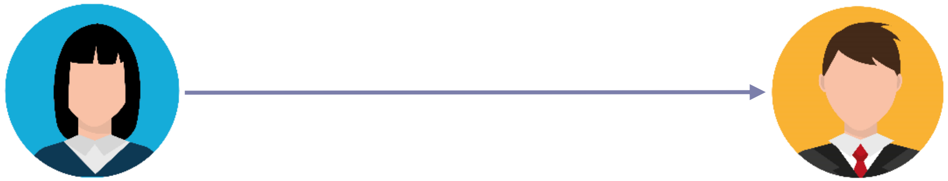
4: **return** $pk = (\mathbf{A}, \mathbf{t})$, $sk = (\mathbf{A}, \mathbf{t}, \mathbf{s}_1, \mathbf{s}_2)$

$$P = \left(P^{[1]}, \dots, P^{[l]}\right)$$

$$P^{[i]} = \sum p_i x^i$$

$$HighBits_q(P^{[i]}, \alpha) = \sum HighBits_q(p_i, \alpha) x^i$$



(A, t, s_1, s_2) (A, t)

$$HighBits_q(P, \alpha) = \left(HighBits_q(P^{[1]}, \alpha), \dots, HighBits_q(P^{[l]}, \alpha)\right)$$

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An overview of ML-DSA

Algorithm Sig

Require: sk, M

Ensure: $\sigma = (c, \mathbf{z})$

```
z = ⊥
while z = ⊥ do
    y ← S̃^l_{γ1}
    w1 := HighBits(Ay, 2γ2)
    c ∈ B_τ := H(M||w1)
    z := y + c s1
    if ||z||_∞ ≥ γ1 − β or LowBits(Ay − c s2, 2γ2)||_∞ ≥ γ2 − β then
        z := ⊥
    end if
end while
return σ = (c, z)
```

$(M, \sigma = (c, \boldsymbol{z}))$

(A, t, s_1, s_2) (A, t)

Alice draws a polynomial vector at random:

$$y \in_R R^l, \quad ||y||_\infty \leq \gamma_1.$$

She computes a random challenge that depends on the message:

$$c = H\left(M \,||\, HighBits_q(Ay, 2\gamma_2)\right).$$

She provides a response to the challenge:

$$z = y + c s_1$$

By definition of z:

$$Az - ct = Ay - c s_2.$$

The signature will be:

$$\sigma = (c, z).$$

But..

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An overview of ML-DSA

Algorithm Sig

Require: sk, M

Ensure: $\sigma = (c, \mathbf{z})$

$\mathbf{z} = \perp$

while $\mathbf{z} = \perp$ **do**

  $\mathbf{y} \leftarrow \tilde{S}^{l}_{\gamma_1}$

  $\mathbf{w}_1 := \mathtt{HighBits}(\mathbf{Ay}, 2\gamma_2)$

  $c \in B_\tau := H(M || \mathbf{w}_1)$

  $\mathbf{z} := \mathbf{y} + c\,\mathbf{s}_1$

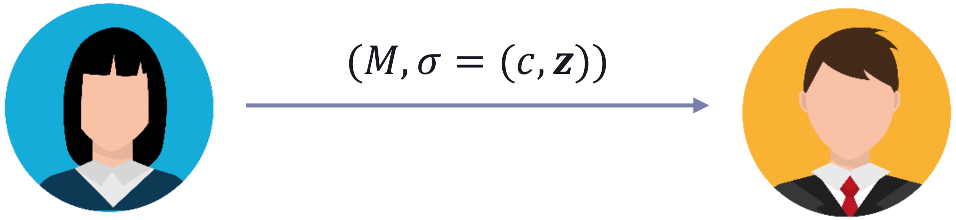
  if $||\mathbf{z}||_\infty \geq \gamma_1 - \beta$ or $\mathtt{LowBits}(\mathbf{Ay} - c\mathbf{s}_2, 2\gamma_2)||_\infty \geq \gamma_2 - \beta$ **then**

    $\mathbf{z} := \perp$

  end if

end while

return $\sigma = (c, \mathbf{z})$



(A, t, s_1, s_2) (A, t)

But..

By definition of z:

$$z = y + cs_1$$

Two conditions must be fulfilled:

$$\begin{cases} ||z||_\infty < max_y\left(||y||_\infty\right) - max_{\{c,s_1\}}\left(||cs_1||_\infty\right) \\ HighBits_q(Ay, 2\gamma_2) = HighBits_q(Ay - cs_2, 2\gamma_2) \end{cases}$$

The first condition is for security, the second for verification and security.

With these conditions:

$$HighBits(Az - ct) = HighBits(Ay - cs_2) = HighBits(Ay)$$

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    z := y + c s1
    if ||z||∞ ≥ γ1 − β or LowBits(Ay − c s2, 2γ2)||∞ ≥ γ2 − β then
        z := ⊥
    end if
end while
return σ = (c, z)
```

Algorithm 1 Ver

```
w'1 := HighBits(Az − ct, 2γ2)
Accept if ||z||∞ ≤ γ1 − β and c = H(M || w'1)
```

Bob can recompute w_1:

$$\begin{aligned} \mathbf{w_1} &= HighBits_q(Ay, 2\gamma_2) \\ &= HighBits_q(Ay - cs_2, 2\gamma_2) \\ &= HighBits_q(Az - ct, 2\gamma_2) \\ &= w'_1 \end{aligned}$$

(A, t, s_1, s_2)

$(M, \sigma = (c, \mathbf{z}))$ →

(A, t)

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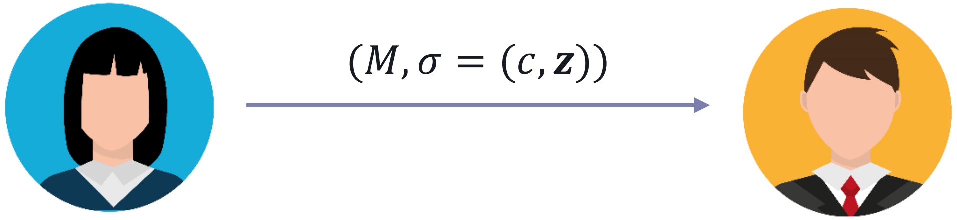
An overview of ML-DSA

Algorithm Sig

Require: sk, M

Ensure: $\sigma = (c, \mathbf{z})$

- $\mathbf{z} = \perp$
- **while** $\mathbf{z} = \perp$ **do**
  - $\mathbf{y} \leftarrow \tilde{S}^{l}_{\gamma_1}$
  - $\mathbf{w}_1 := \texttt{HighBits}(\mathbf{A}\mathbf{y}, 2\gamma_2)$
  - $c \in B_\tau := H(M \| \mathbf{w}_1)$
  - $\mathbf{z} := \mathbf{y} + c\,\mathbf{s}_1$
  - **if** $\|\mathbf{z}\|_\infty \geq \gamma_1 - \beta$ or $\|\texttt{LowBits}(\mathbf{A}\mathbf{y} - c\mathbf{s}_2, 2\gamma_2)\|_\infty \geq \gamma_2 - \beta$ **then**
    - $\mathbf{z} := \perp$
  - **end if**
- **end while**
- **return** $\sigma = (c, \mathbf{z})$



(A, t, s_1, s_2) $\quad (M, \sigma = (c, \mathbf{z})) \quad$ (A, t)

Algorithm 1 Ver

- $\mathbf{w}_1' := \texttt{HighBits}(\mathbf{A}\mathbf{z} - c\mathbf{t}, 2\gamma_2)$
- **Accept if** $\|\mathbf{z}\|_\infty \leq \gamma_1 - \beta$ **and** $c = H(M \| \mathbf{w}_1')$

Bob can recompute w_1:

$$\begin{aligned} \mathbf{w}_1 &= HighBits_q(Ay, 2\gamma_2) \\ &= HighBits_q(Ay - cs_2, 2\gamma_2) \\ &= HighBits_q(Az - ct, 2\gamma_2) \\ &= w_1' \end{aligned}$$

All that aside, the most important relation is:

$$z = y + cs_1$$

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Existing fault attack on ML-DSA

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A fault attack on ML-DSA

Loop-Abort Faults on Lattice-Based Fiat–Shamir and Hash-and-Sign Signatures

Thomas Espitau[4], Pierre-Alain Fouque[2], Benoît Gérard[1], and Mehdi Tibouchi[3]

[EFGT17]: Published at SAC2017 and describes a fault attack against BLISS.

Algorithm 1 Sig

Require: sk, M

Ensure: $\sigma = (c, \mathbf{z})$

- $\mathbf{z} = \perp$
- **while** $\mathbf{z} = \perp$ **do**
  - $\mathbf{y} \leftarrow \tilde{S}^{l}_{\gamma_1}$
  - $\mathbf{w}_1 := \texttt{HighBits}(\mathbf{A}\mathbf{y}, 2\gamma_2)$
  - $c \in B_\tau := H(M \| \mathbf{w}_1)$
  - $\mathbf{z} := \mathbf{y} + c\,\mathbf{s}_1$
  - **if** $\|\mathbf{z}\|_\infty \geq \gamma_1 - \beta$ or $\texttt{LowBits}(\mathbf{A}\mathbf{y} - c\mathbf{s}_2, 2\gamma_2)\|_\infty \geq \gamma_2 - \beta$ **then**
    - $\mathbf{z} := \perp$
  - **end if**
- **end while**
- **return** $\sigma = (c, \mathbf{z})$

Main Idea: Inject a fault to obtain one of the coefficients of $\mathbf{y}$ of abnormally small degree.

They consider a signature $\sigma = (c, z)$ with

$$z^{[1]} = y^{[1]} + c s_1^{[1]} \quad \text{and} \quad \deg\left(y^{[1]}\right) = m \ll n$$

This will make $s_1{}^{[1]}$ the smallest vector in a lattice of sufficiently small dimension to find it.

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A fault attack on ML-DSA

Single fault attack:

One has:

$$z^{[1]} = y^{[1]} + cs_1^{[1]}$$

Thus if c is invertible:

$$s_1^{[1]} = c^{-1}z^{[1]} - \sum_{i=0}^{m} y_i^{[1]}(cx)^i \; mod(q).$$

Therefore,

$$s_1^{[1]} \in L(c^{-1}z^{[1]}, \{ (cx)^i \}_{i\in\{0,\dots,m\}})$$

If m is sufficently small, $s_1^{[1]}$ can be recovered using lattice reduction technique (LLL or BKZ).

Practical results:

Fault after iteration number $m =$	20	40	60	80	100
Theoretical minimum dimension $\ell_{\min}$	22	44	66	88	110
Dimension ℓ in our experiment	24	50	80	110	140
Lattice reduction algorithm	LLL	BKZ–20	BKZ–25	BKZ–25	BKZ–25
Success probability (%)	100	100	100	100	—
Avg. CPU time to recover ℓ coeffs. (s)	0.23	7.3	119	941	10500
Avg. CPU time for full key recovery	5 s	80 s	14 min	80 min	12 h

Conclusion:

The fault attack is plausible.

The fault needs to be injected before the generation of the 100 first coefficients.

Proposed countermeasure: Shuffling the order of the coefficient's generation.

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Our results

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A fault attack on ML-DSA

Limitations:

- Less realistic for ML-DSA.
- Simple countermeasures.
- Single fault attack?

Our questions:

- Applicable to ML-DSA?
- Possible to improve with more faults?
- Possible to overcome the simple countermeasure?
- Turn it into a passive attack?

Algorithm 34 ExpandMask(ρ, μ)

Samples a vector $\mathbf{y} \in R^\ell$ such that each polynomial $\mathbf{y}[r]$ has coefficients between $-\gamma_1 + 1$ and γ_1.

Input: A seed $\rho \in \mathbb{B}^{64}$ and a nonnegative integer μ.
Output: Vector $\mathbf{y} \in R^\ell$.

```
1: c ← 1 + bitlen (γ1 − 1)                      ▷ γ1 is always a power of 2
2: for r from 0 to ℓ − 1 do
3:     ρ′ ← ρ||IntegerToBytes(μ + r, 2)
4:     v ← H(ρ′, 32c)                            ▷ seed depends on μ + r
5:     y[r] ← BitUnpack(v, γ1 − 1, γ1)
6: end for
7: return y
```

Algorithm 19 BitUnpack(v, a, b)

Reverses the procedure BitPack.

Input: $a, b \in \mathbb{N}$ and a byte string v of length $32 \cdot \text{bitlen}\ (a+b)$.
Output: A polynomial $w \in R$ with coefficients in $[b - 2^c + 1, b]$, where $c = \text{bitlen}\ (a+b)$.
When $a + b + 1$ is a power of 2, the coefficients are in $[-a, b]$.

```
1: c ← bitlen (a + b)
2: z ← BytesToBits(v)
3: for i from 0 to 255 do
4:     w_i ← b − BitsToInteger((z[ic], z[ic + 1], ... z[ic + c − 1]), c)
5: end for
6: return w
```


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A fault attack on ML-DSA: Improvement

Let $\sigma_1, \dots, \sigma_m$ be m signatures such that:

$$\forall i \in \{1, \dots, m\}, z_i^{[1]} = y_i^{[1]} + c s_1^{[1]} \text{ with } \deg\left(y_i^{[1]}\right) \leq d < n - 1.$$

Then one can construct m lattices such that:

$$\forall i \in \{1, \dots, m\}, \quad \dim(L_i) \leq d + 2 \quad \text{and} \quad s_1^{[1]} \in L = \bigcap L_i$$

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Formally:

$$\forall i \in \{1, \dots, m\}, \qquad L_i = L\left(c_i^{-1} z_i^{[1]}, \{ (c_i x)^j \}_{j \in \{0, \dots, d\}}\right) \quad and \quad L = \bigcap L_i$$

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Formally:

$$\forall i \in \{1, \dots, m\}, \quad L_i = L\left(c_i^{-1} z_i^{[1]}, \{ (c_i x)^j \}_{j \in \{0, \dots, d\}}\right) \quad and \quad L = \bigcap L_i$$

We have reformulated the problem of finding $s_1^{[1]}$, as the calculation of a lattice intersection

To have $d + 2 < n$, one needs $d \leq n - 3$.

The attack requires knowledge of 2 coefficients.

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A fault attack on ML-DSA: How to intersect lattices efficently?

Classic method: Using duality

Let $L_1 = L(B_1)$ and $L_2 = L(B_2)$ be two lattices.

Union: $L_1 \cup L_2 = L\big(HNF(B_1|B_2)\big)$ and Duality relation: $(L_1 \cup L_2)^* = L_1^* \cap L_2^*$

Lead to:

$$L_1 \cap L_2 = \Big(L\big(HNF(D_1|D_2)\big)\Big)^*,$$

with D_1, D_2 such that $L_1^* = L(D_1)$ and $L_2^* = L(D_2)$.

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with D_1, D_2 such that $L_1^* = L(D_1)$ and $L_2^* = L(D_2)$.

Problems: For $L \subset \mathbb{Z}^n$ generally $L^* \not\subset \mathbb{Z}^n$. One have to compute HNF over $\mathbb{Q}$, and numerators and denominators explode. This leads to rounding errors when calculating the HNF and an explosion in calculation time.

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A fault attack on ML-DSA: How to intersect lattices efficently?

Optimized method: Using $\mathbb{F}_q$-subspaces.

Let $L_1 = L(B_1)$ and $L_2 = L(B_2)$ be two lattices, such that $L_1, L_2 \subset q\mathbb{Z}^n$

1. View $\overline{L_1}, \overline{L_2}$ as $\mathbb{F}_q$-subspaces
2. Compute an intersection of subspaces: $\overline{L} = \overline{L_1} \cap \overline{L_2}$ and B a basis of $\overline{L}$.
3. View $\overline{L}$ as an integer lattice by considering: $L = L\left(B, \{qx^j\}_{j\in\{0,\ldots,n-1\}}\right)$

Solution: No need to work in rationnal field. Better complexity.

1. Attack can be improved with more faults
2. No restriction on fault injection at the time of y generation

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1. View $\overline{\mathrm{L}_1}, \overline{L_2}$ as $\mathbb{F}_q$-subspaces
2. Compute an intersection of subspaces: $\bar{L} = \overline{L_1} \cap \overline{L_2}$ and B a basis of $\bar{L}$.
3. View $\bar{L}$ as an integer lattice by considering: $L = L\left(B, \{qx^j\}_{j\in\{0,\dots,n-1\}}\right)$

Solution: No need to work in rationnal field. Better complexity.

1. Attack can be improved with more faults
2. No restriction on fault injection at the time of y generation

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A fault attack on ML-DSA: How to intersect lattices efficently?

Optimized method: Using $\mathbb{F}_q$-subspaces.

Let $L_1 = L(B_1)$ and $L_2 = L(B_2)$ be two lattices, such that $L_1, L_2 \subset q\mathbb{Z}^n$

1. View $\overline{\mathrm{L}_1}, \overline{L_2}$ as $\mathbb{F}_q$-subspaces
2. Compute an intersection of subspaces: $\overline{L} = \overline{L_1} \cap \overline{L_2}$ and B a basis of $\overline{L}$.
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But how do you turn it into a passive attack?

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A fault attack on ML-DSA: Considering affine lattices

To switch from a fault-based attack to a side channel attack, the attack must operate with a single coefficient.

But:

$$\dim\left(L_i = L\left(c_i^{-1} z_i^{[1]}, \{\,(c_i x)^j\,\}_{j\in\{0,\dots,d\}}\right)\right) = d + 2.$$

To have $d + 2 < n$, one needs $d \leq n - 3$.

The attack requires knowledge of 2 coefficients.

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A fault attack on ML-DSA: Considering affine lattices

Easy fix: By considering affine lattices,

$$\forall i \in \{1, \dots, m\}, \qquad A_i = c_i^{-1} z_i^{[1]} + L\left(\{\, (c_i x)^j \,\}_{j \in \{0, \dots, d\}}\right) \;\; and \;\; A = \bigcap A_i$$

This time, $\dim(A_i) = d + 1$. We simply need to adapt the attack to the affine case:

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A fault attack on ML-DSA: Considering affine lattices

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This time, $\dim(A_i) = d + 1$. We simply need to adapt the attack to the affine case:

$L_i = L\left(c_i^{-1} z_i^{[1]}, \left\{ (c_i x)^j \right\}_{j \in \{0, \dots, d\}}\right) \ \ and \ \ L = \bigcap L_i$	$A_i = c_i^{-1} z_i^{[1]} + L\left(\left\{ (c_i x)^j \right\}_{j \in \{0, \dots, d\}}\right) \ \ and \ \ A = \bigcap A_i$
Computing $\bar{L} = \bigcap \bar{L_i}$	Computing $\bar{A} = \bigcap \bar{A_i}$
Using LLL	Using Babai's NPA

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Practical results

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A fault attack on ML-DSA: Results

d	20	40	60	80	90
Theoretical l_{min}	27	53	79	105	118
l in practice	27	53	79	115	188
Probability of success	1	1	1	1	4/5
recover $\mathbf{s}_1^{[1]}$	0.272s	2.65s	13.69s	60.49s	866.9s
used algorithm	LLL	BKZ25	BKZ25	BKZ30	BKZ30

Attack results with a single signature against ML-DSA-II

m	220	200	180	160
$\dim_{\mathbb{F}_q}(L)$	36	56	76	96
Theoretical l_{min}	41	64	87	109
l in practice	50	65	90	120
Success for our l	1	1	1	1
recover $\mathbf{s}_1^{[1]}$	89.38s	84.27s	84.9s	1744.9s
used algorithm	LLL	BKZ25	BKZ25	BKZ30

Attack results with m signatures against ML-DSA-II

- **The attack is applicable to ML-DSA and more effective with a few faults.**
- **The suggested countermeasure is not sufficient.**

- **If the attacker knows a single coefficient, he needs 160 signatures to find the secret key.**

The code is publicly available: [GitHub - AzevedoPaco/AttackML-DSA]

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Thank you

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