Accelerating Post-quantum Secure zkSNARKs by Optimizing Additive FFT

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Selected Areas in Cryptography (SAC) 2025
Toronto, Canada
(13 – 15 August 2025)

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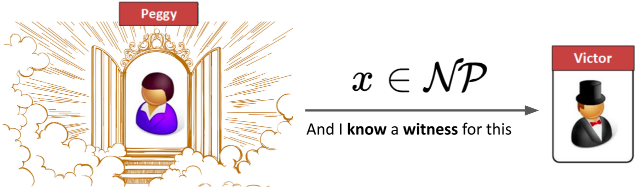
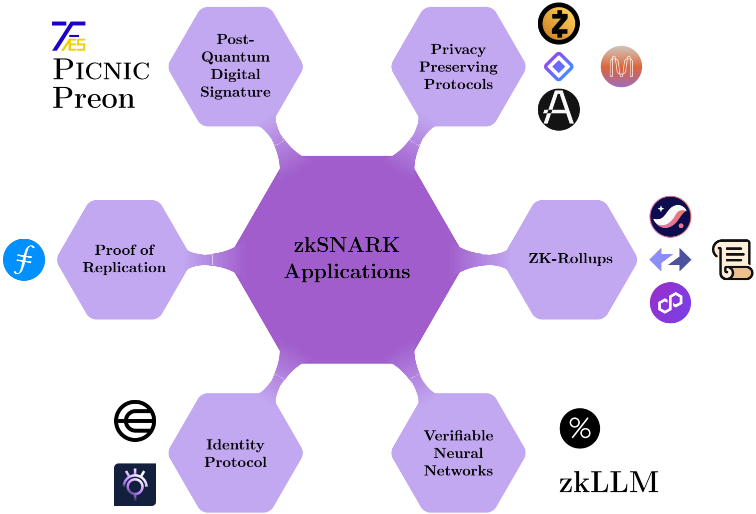
zkSNARK: Zero-Knowledge Succinct Non-interactive Argument of Knowledge

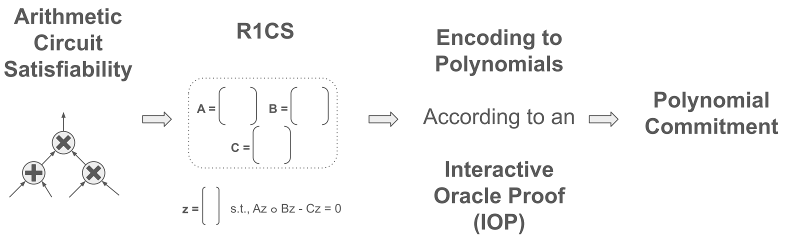


- Peggy (Prover) creates a **non-interactive (publicly verifiable)** argument about $x \in \mathcal{NP}$ and she **knows** a witness w for this.
- The argument convinces Victor (Verifier) that $x \in \mathcal{NP}$ but does not reveal any knowledge about w.

Applications of zkSNARKs



A Brief Background on zkSNARKs' Construction



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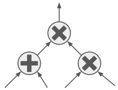
Arithmetic Circuit Satisfiability



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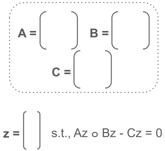
R1CS



⇒

Encoding to Polynomials

According to an

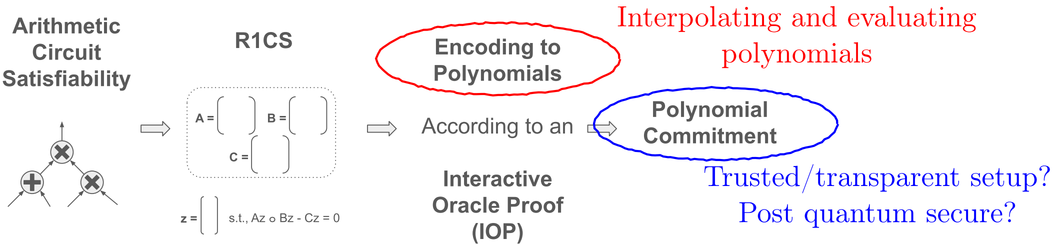
Interactive Oracle Proof (IOP)

⇒

Polynomial Commitment

Interpolating and evaluating polynomials

A Brief Background on zkSNARKs' Construction



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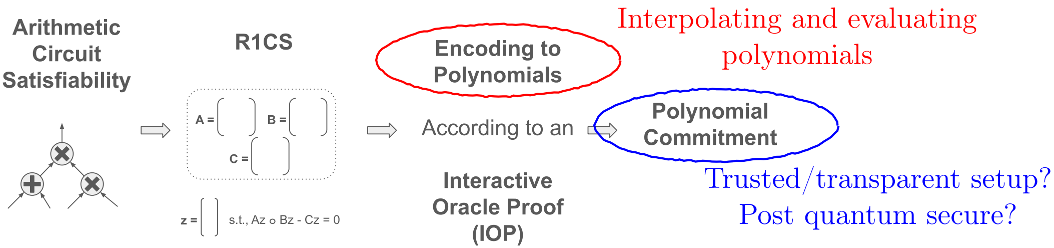

Example:
Fast Reed-Solomon IOP (FRI) + Merkle Hash Tree (MHT) Commitment
(Ben-Sasson et al., 2018)

is a core component of many transparent setup post-quantum secure zkSNARKs.

Core Assumptions and Domains in Modern zkSNARKs

zkSNARK	Cryptographic Assumption	Algebraic Domain
Groth16	KoE	elliptic-curve group
PLONK	KoE	elliptic-curve group
HALO	ECDLP	elliptic-curve group
Aurora	CRH	any algebraic domain
STARK	CRH	any algebraic domain
Fractal	CRH	any algebraic domain
Ligero	CRH	any algebraic domain

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FRI-based	Fractal	CRH	any algebraic domain
	Ligero	CRH	any algebraic domain

Challenges and Limitations in FRI-based zkSNARKs and Ligero

- High prover / verifier time complexity (addressed in this paper)

Polygon ZK-EVM	Generates a proof for 27 transactions on 128 vCPUs and 1024 GB RAM (Chaliasos et al., 2024).	**311 seconds**
Preon (Post-Quantum Signature)	Creates one signature (256-bit security) on a single-core AMD EPYC 73F3 @ 3.5 GHz (NIST PQC Round 1: Additional Signatures).	**417 seconds**

- Large argument (proof) size
- etc.

Main Contributions

1. **Acceleration of Aurora (Ben-Sasson et al., 2019) zkSNARK**: Optimized polynomial evaluation and interpolation via additive FFT improvements.

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2. **Cantor additive FFT:**
   - Exact finite field operation counts.
   - Precomputation techniques and memory–runtime trade-offs, with extra savings for structured subspaces.
   - Applied in Aurora to measure acceleration gains.

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   - Reduced finite field operation counts using Cantor special basis.
   - Precomputation techniques and memory–runtime trade-offs.
4. **LCH additive FFT:**
   - Compared with other FFTs when LCH uses the Cantor special basis.
   - Measured acceleration gains when integrated into Aurora.

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Discrete Fourier Transform (DFT)

Let $W = \{\eta_0, \eta_1, \ldots, \eta_{n-1}\}$ be the evaluation set.

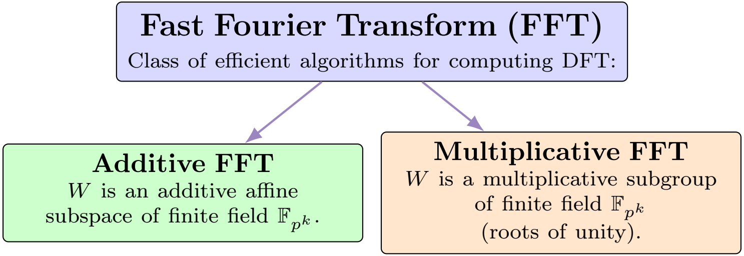
$$\textbf{DFT: } f(x) = c_0 + c_1 x + \cdots + c_{n-1}x^{n-1} \xrightarrow{\text{evaluation}} \begin{bmatrix} f(\eta_0) \\ f(\eta_1) \\ \vdots \\ f(\eta_{n-1}) \end{bmatrix}$$

$$\textbf{IDFT: } \begin{bmatrix} (\eta_0, f(\eta_0)) \\ (\eta_1, f(\eta_1)) \\ \vdots \\ (\eta_{n-1}, f(\eta_{n-1})) \end{bmatrix} \xrightarrow{\text{interpolation}} f(x) = c_0 + c_1 x + \cdots + c_{n-1}x^{n-1}$$

The prover algorithm in **Aurora** performs $\boxed{7\lambda_i + \lambda_i' + 10}$ polynomial evaluations and interpolations, where λ_i and λ_i' are repetition parameters [1].

[1] The number of times of calling lincheck and FRI-LDT subprotocols.

Fast Fourier Transform (FFT)



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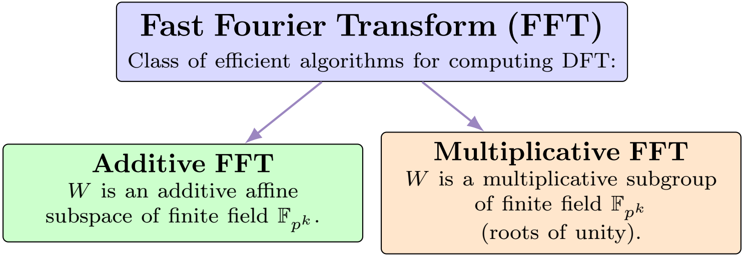

- The structure of W is dictated by the *polynomial commitment scheme* in zkSNARKs.
- **FRI+MHT** supports both additive affine subspaces (of binary fields) and multiplicative subgroups (of prime fields).

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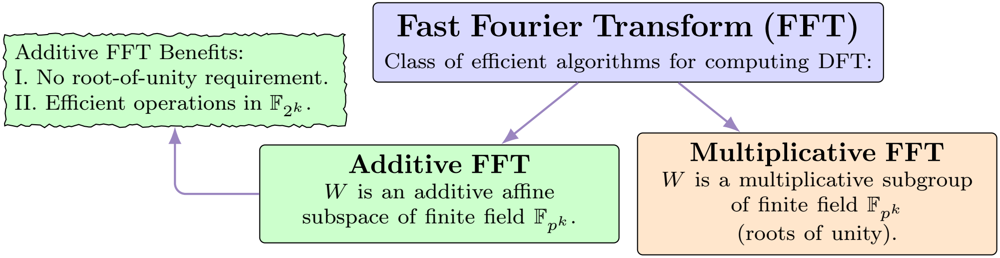

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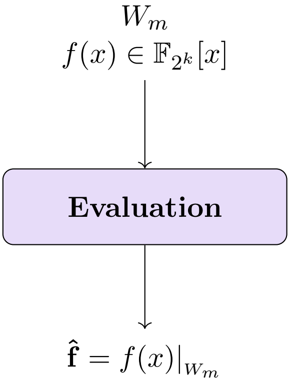
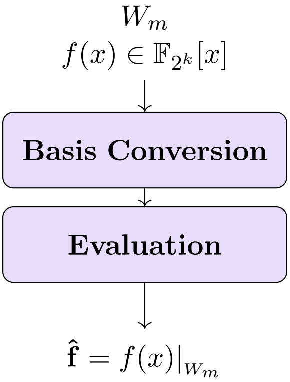
Additive FFT Algorithms

- Cantor (1989)
- von zur Gathen and Gerhard (1996)



- Gao and Mateer (2010)
- Lin, Chung, and Han (2014)



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Cantor Additive FFT Algorithm (Cantor, 1989)

- $\mathbb{Z}_{W_i}(x)$ denotes a vanishing polynomial over $W_i := \langle \beta_0, \beta_1, \ldots, \beta_i \rangle$.
- $\mathbb{Z}_{W_i}(x)$ is a linearized polynomial $\rightarrow$ $\mathbb{Z}_{W_i}(x + \theta) = \mathbb{Z}_{W_i}(x) + \mathbb{Z}_{W_i}(\theta)$

$$W_m = \langle \beta_0, \beta_1, \ldots, \beta_{m-1} \rangle, \beta_i \in \mathbb{F}_{2^k}$$

$$f(x) \in \mathbb{F}_{2^k}[x], \deg(f) < 2^m$$

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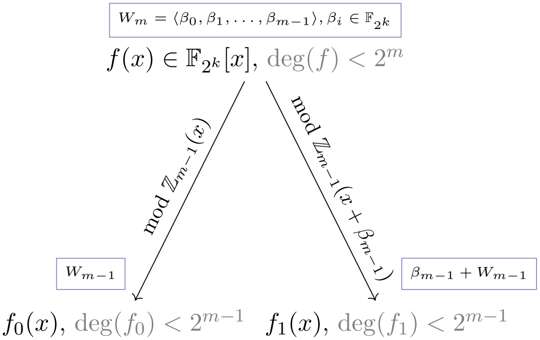

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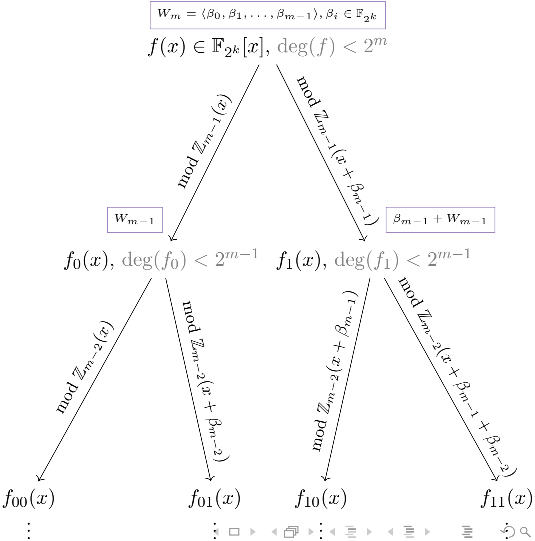

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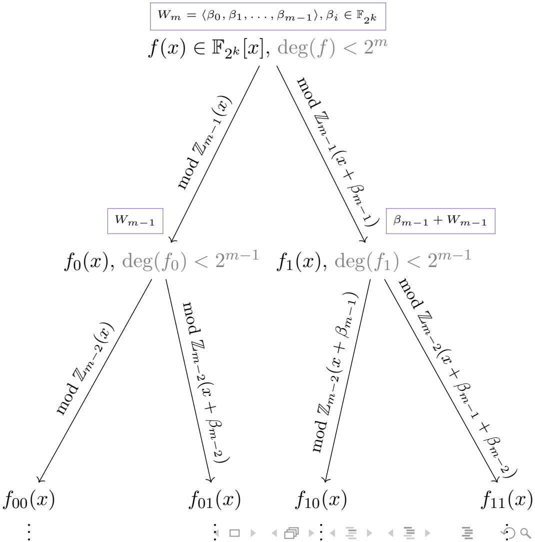
- For $f(x) \in \mathbb{F}_{2^k}[x]$, with $k = t \cdot 2^\ell$ $(2^\ell \geq m)$ $\{\beta_0, \beta_1, \ldots, \beta_{m-1}\}$ denotes Cantor special basis if $\boxed{\beta_{i-1} = \beta_i^2 + \beta_i \text{ and } \beta_0 = 1}$.



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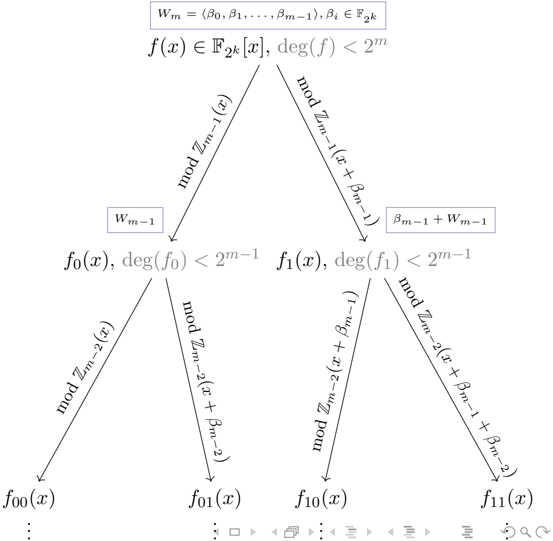

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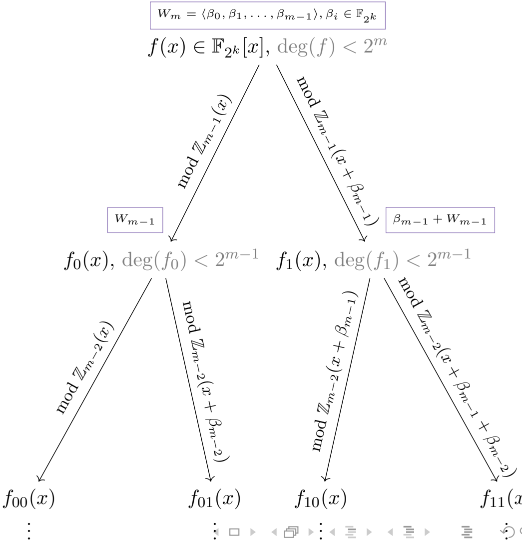
    - $\mathbb{Z}_{W_i}(\beta_{i+\ell}) = \beta_\ell \rightarrow \mathbb{Z}_{W_i}(\beta_i) = 1$
    - $\mathbb{Z}_{W_i}(x) \in \mathbb{F}_2[x]$



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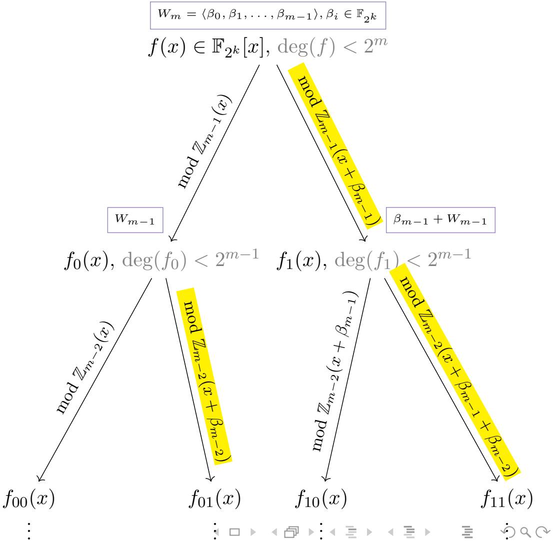
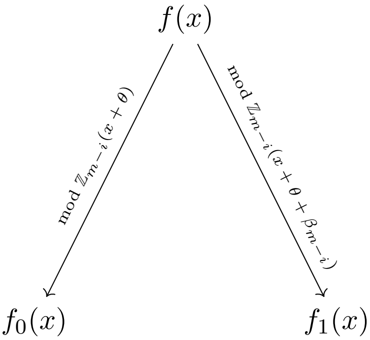
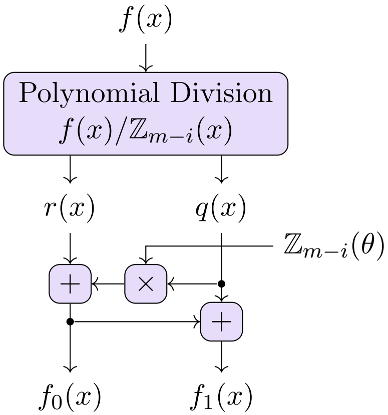


Cantor Additive FFT: Polynomial Division Algorithm



$$f(x) \in \mathbb{F}_{2^k}[x],$$
$$\deg(f) < 2^{m-i+1}$$

$$\equiv$$



$$f_0(x) = r(x) + \mathbb{Z}_{W_{m-i}}(\theta)\, q(x),$$
$$f_1(x) = f_0(x) + q(x)$$

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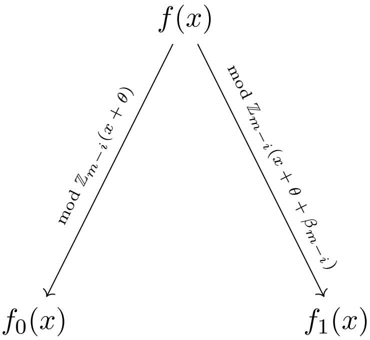

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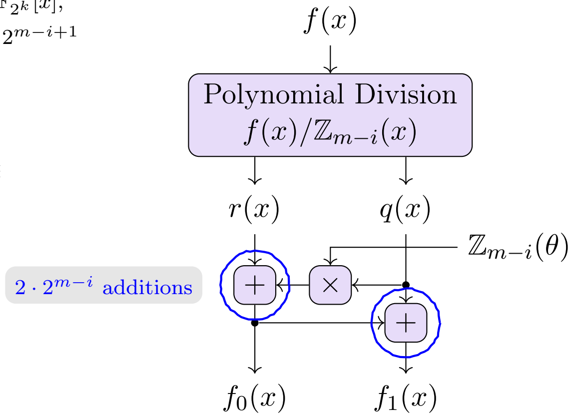

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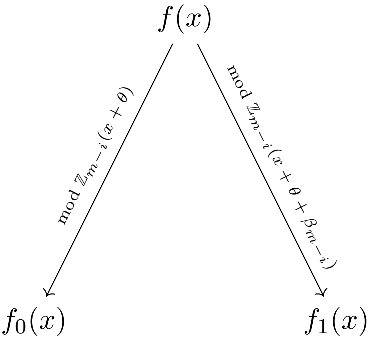

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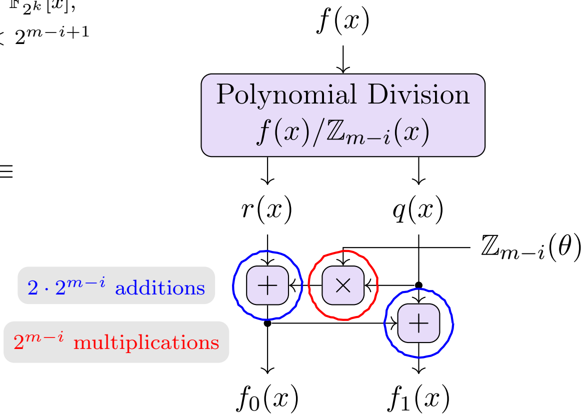

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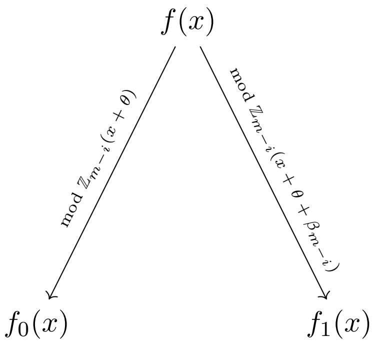


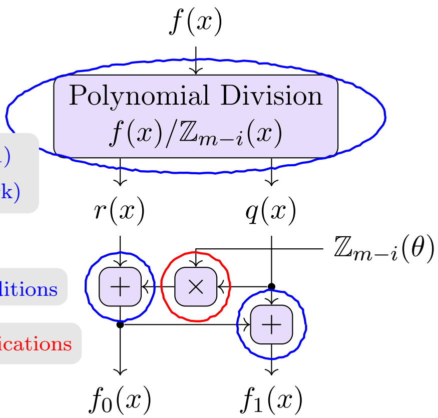
$$f_0(x) = r(x) + \mathbb{Z}_{W_{m-i}}(\theta)\, q(x),$$
$$f_1(x) = f_0(x) + q(x)$$

$f(x) \in \mathbb{F}_{2^k}[x]$,
$\deg(f) < 2^{m-i+1}$

$2^{m-i}(2^{\mathrm{wt}(m-i)} - 1)$ additions (This work)

$2 \cdot 2^{m-i}$ additions

2^{m-i} multiplications

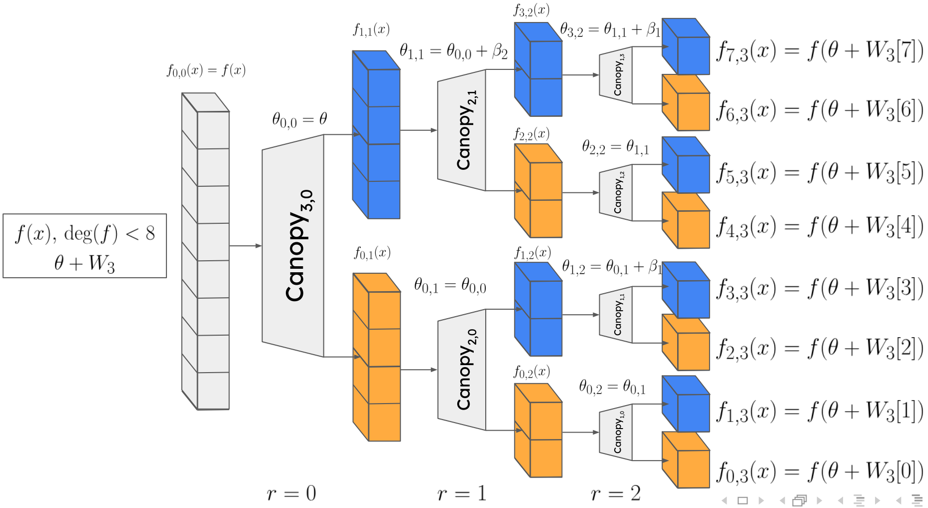


Cantor Additive FFT: Finite Field Operations

The exact number of finite field additions and multiplications in the Cantor additive FFT

- Additions: $\frac{1}{2} n \log_2 n + \frac{1}{2} n \sum_{r=0}^{\log_2(n)-1} 2^{\mathrm{wt}(r)}$
- Multiplications: $\frac{1}{2} n \log_2 n$

Cantor Additive FFT: Final Structure



Cantor Additive FFT: Precomputation

- $\mathbb{Z}_{W_{i,r}}(\theta_{i,r})$ can be precomputted for any predetermined $\theta + W_m$
- This requires $2^m - 1$ elements in $\mathbb{F}_{2^k}$.

Special Case $\theta \in \{\beta_0, \beta_1, \ldots, \beta_{b-1}\}$:

- It is only required to compute $\langle \beta_0, \beta_1, \ldots, \beta_{b-1} \rangle$ for any FFT of length $2^m < 2^b$.
- Multiple lookup tables:

$$\langle \beta_0, \ldots, \beta_{\frac{b}{\ell}-1} \rangle, \langle \beta_{\frac{b}{\ell}}, \ldots, \beta_{2\frac{b}{\ell}-1} \rangle, \ldots, \langle \beta_{(\ell-1)\frac{b}{\ell}}, \ldots, \beta_{\ell\frac{b}{\ell}-1} \rangle$$

which requires $\ell \cdot 2^{\frac{b}{\ell}}$ elements in $\mathbb{F}_{2^k}$.

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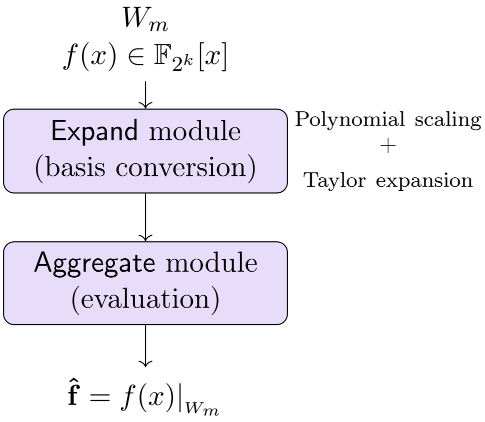
Gao–Mateer Additive FFT (Gao and Mateer, 2010)

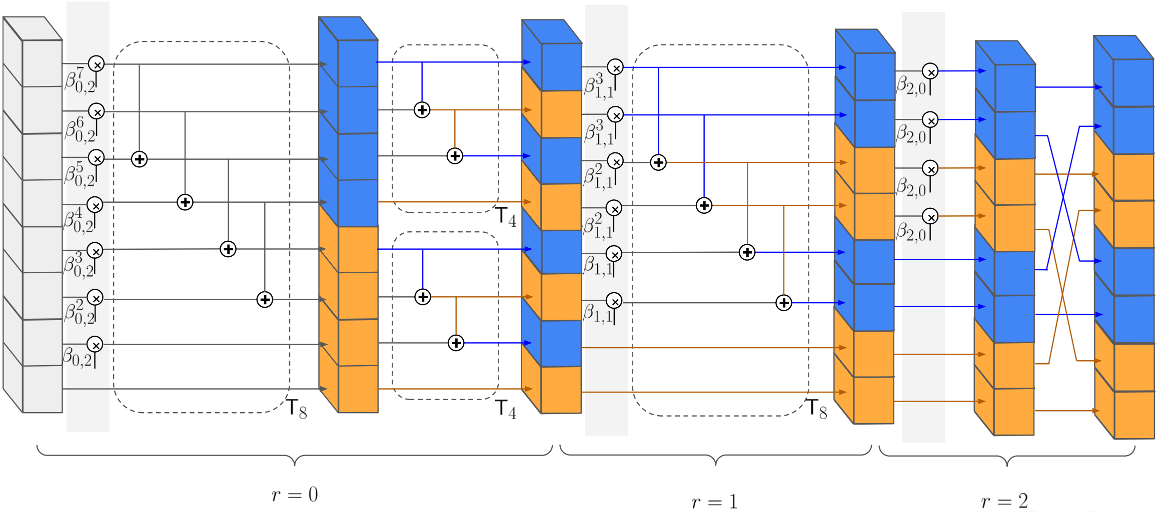
- Applicable to $f(x) \in \mathbb{F}_{2^k}[x]$ for any arbitrary k.



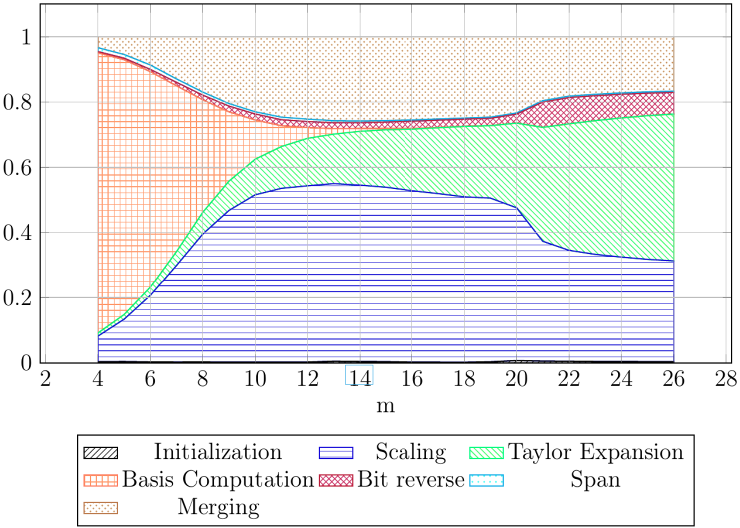
Optimizations:

- Precomputation:
  - Level 1: Basis elements for each round of the Expand and Aggregate module.
  - Level 2: All multiplication factors in the both modules.
- Using the Cantor Special Basis:
  - Saves $n \log_2 n - n + 1$ multiplications in the Expand module.

Gao–Mateer Additve FFT Optimization: Expand Module



Gao–Mateer Additve FFT: Cost of Each Component



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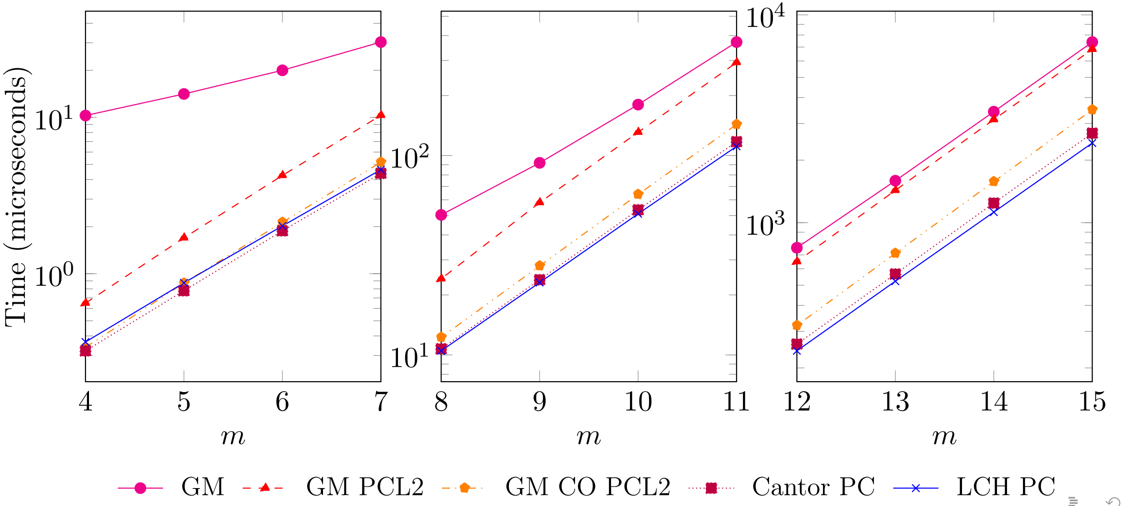
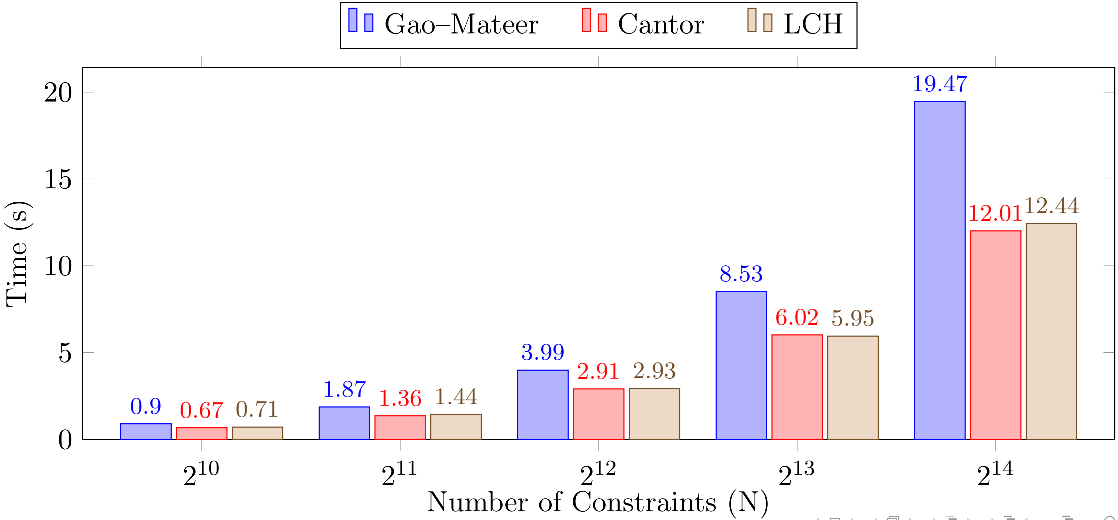
Finitie Field Operation Comparison

FFT	Basis	Basis Conversion	Evaluation
GM	General	#A: $\frac{1}{4}n(\log_2 n)^2 - \frac{1}{4}n\log_2 n$ #M: $n\log_2 n - n + 1$	#A: $n\log_2 n$ #M: $\frac{1}{2}n\log_2 n$
	Cantor	#A: $\frac{1}{4}n(\log_2 n)^2 - \frac{1}{4}n\log_2 n$ #M: 0	#A: $n\log_2 n$ #M: $\frac{1}{2}n\log_2 n$
LCH	General (Lin et al., 2016)	#A: $O(n(\log_2 n)^2)$ #M: $O(n\log_2 n)$	#A: $n\log_2 n$ #M: $\frac{1}{2}n\log_2 n$
	Cantor (Lin et al., 2016)	#A: $O(n\log_2 n\log_2\log_2 n)$ #M: 0	#A: $n\log_2 n$ #M: $\frac{1}{2}n\log_2 n$
Cantor	Cantor	N/A	#A: $\frac{1}{2}n\log_2 n + \frac{1}{2}n\sum_{r=0}^{\log_2(n)-1} 2^{\mathrm{wt}(r)}$ #M: $\frac{1}{2}n\log_2 n$

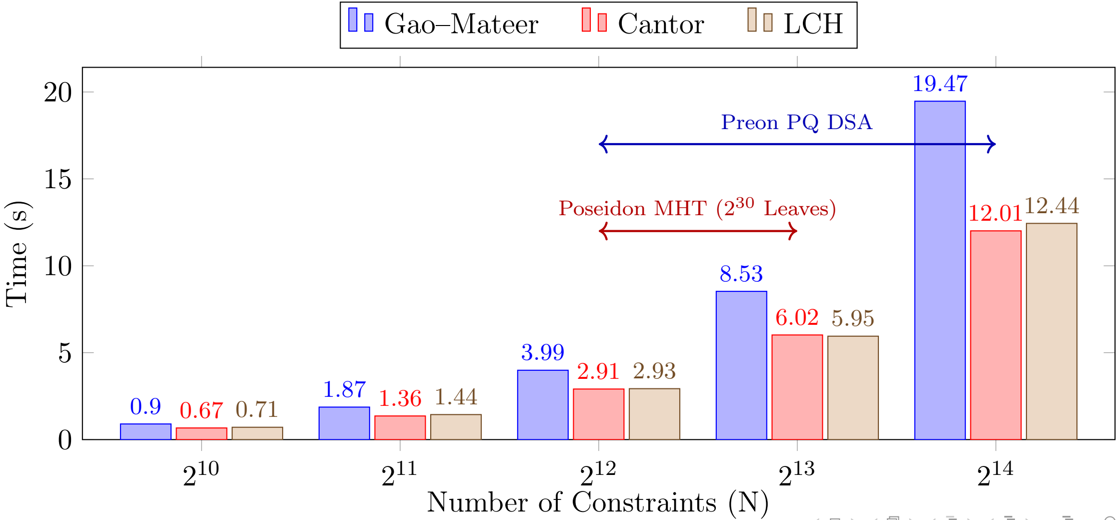
Additive FFT Benchmark (input length: $n = 2^m$)



Aurora Prover Timing Comparison Across FFT Implementations



Aurora Prover Timing Comparison Across FFT Implementations



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- We proposed precomputation techniques that reduce overhead for both Cantor and Gao–Mateer FFTs when the affine subspace basis is fixed.

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- Side-channel analysis of additive FFT implementations.

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Thank You!

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Any questions?