

Collision Attacks on Spongnet with Grouping Method

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Contents

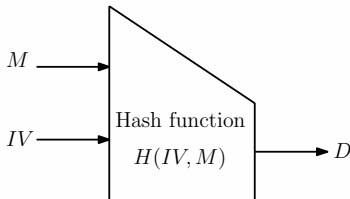
- 1. Motivation & Background**
2. Our Method
3. Application to Collision Attacks
4. Conclusion

■ Functionality

- Hash functions map an arbitrary-length input to a fixed-length digest

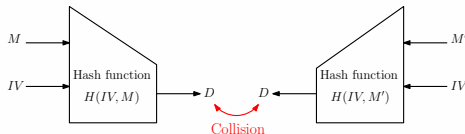
■ Security requirements

- Collision resistance
- Preimage resistance
- Second-preimage resistance



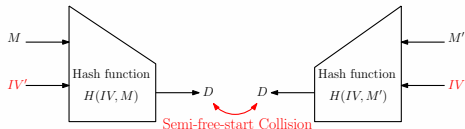
Collision Attacks

- Find M and M' such that $H(IV, M) = H(IV, M')$ for a pre-fixed IV



Semi-free-start Collision Attacks

- Find M , M' , and IV' such that $H(IV', M) = H(IV', M')$

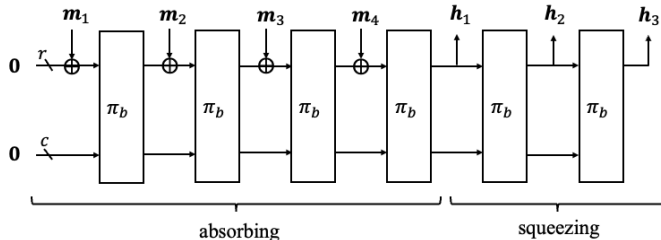


Free-start Collision Attacks

- Find M , M' , IV'_0 , and IV'_1 such that $H(IV'_0, M) = H(IV'_1, M')$

SPONGENT

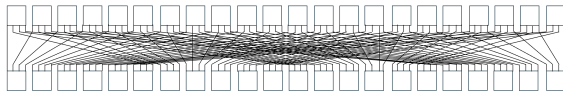
- Proposed by Bogdanov et al. at CHES 2011
- Based on sponge construction with PRESENT-like permutations [BKL⁺07]
- Comprises 13 variants, denoted as SPONGENT- $n/c/r$,
 - ✓ n : hash size, c : capacity, r : rate, $r + c$: block size



1.4 SPONGENT: Underlying Permutations

■ PRESENT-like permutations [BKL⁺07]

- Based on Substitution-Permutation Network
- Consist of a 4-bit S-box and a bit-wise permutation



The round function of SPONGENT-88/80/8

- sBoxLayer
 - ✓ Apply the 4-bit S-box in parallel

$$S[\cdot] = \{0xE, 0xD, 0xB, 0x0, 0x2, 0x1, 0x4, 0xF, 0x7, 0xA, 0x8, 0x5, 0x9, 0xC, 0x3, 0x6\}$$

- pLayer
 - ✓ Apply a b -bit permutation

$$P_b(j) = \begin{cases} j \cdot b/4 \bmod (b-1), & \text{if } j \in \{0, \dots, b-2\} \\ b-1, & \text{if } j = b-1. \end{cases}$$

■ Claimed security for collision attacks

	n	b	c	r	Rounds	Security (bit)		n	b	c	r	Rounds	Security (bit)	
SPONGENT-88/80/8	88	88	80	8	45	40		SPONGENT-224/224/16	224	240	224	16	120	112
SPONGENT-88/176/88	88	264	176	88	135	44		SPONGENT-224/224/112	224	336	224	112	170	112
SPONGENT-128/128/8	128	136	128	8	70	64		SPONGENT-224/448/224	224	672	448	224	340	112
SPONGENT-128/256/128	128	384	256	128	195	64		SPONGENT-256/256/16	256	272	256	16	140	128
SPONGENT-160/160/16	160	176	160	16	90	80		SPONGENT-256/256/128	256	384	256	128	195	128
SPONGENT-160/160/80	160	240	160	80	120	80		SPONGENT-256/512/256	256	768	512	256	385	128
SPONGENT-160/320/160	160	480	320	160	240	80								

■ Previous results

- The designers demonstrated a rebound attack on 6-round SPONGENT-88/80/8
 - ✓ Time complexity is $2^{55.2} (> 2^{40})$
- Abdelraheem demonstrated the distinguishing attacks based on differential and linear characteristics [Abd12]
- Zhang and Liu demonstrated the distinguishing attacks based on truncated differentials with a MITM technique [ZL17]
- **No third-party security analysis on collision resistance has been reported**

■ Difficulty in obtaining sufficient degrees of freedom (DoF)

- For block-cipher-based hash functions, this can be done by counting the available DoF in each S-box independently, such as in the super-S-box technique used in rebound attacks [GP10].
- For public-permutation-based hash functions, we need to consider the dependency across all S-boxes in all rounds
 - ✓ Making it much more complex to obtain sufficient DoF

■ Automatic search method proposed by Liu et al. [LIM20].

- Finding a valid differential characteristic for Gimli
 - ✓ They showed that many differential characteristics in previous studies on sponge-based hash functions are invalid

We extend Liu et al.'s method to obtain sufficient DoF for public-permutation-based hash functions

■ First third-party security analysis for collision resistance of SPONGENT

➤ * indicates that the real colliding pair was found

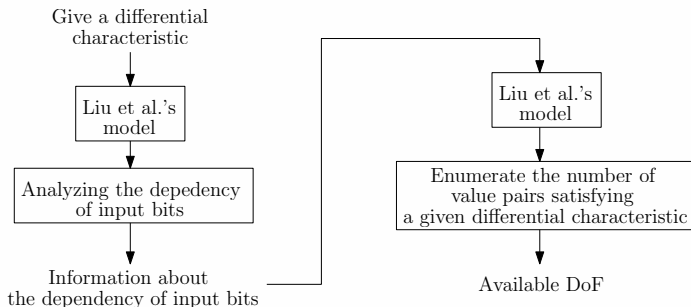
	Collision		Semi-free-start		Ref	Full round	Claimed security
	Rounds	Time	Rounds	Time			
SPONGENT-88/80/8	-	-	8*	2^{17}	Ours	45	2^{40}
	6†	$2^{55.2}$	-	-	[BKL ⁺ 11]		
SPONGENT-88/176/88	9 8*	2^{42} 2^{28}	9*	2^{33}	Ours	135	2^{44}
SPONGENT-128/128/8	-	-	8*	2^{22}	Ours	70	2^{64}
SPONGENT-128/256/128	12 8*	2^{58} 2^{28}	14	2^{63}	Ours	195	2^{64}
SPONGENT-160/160/16	-	-	7	2^{47}	Ours	90	2^{80}
SPONGENT-160/160/80	11	2^{61}	14	2^{78}	Ours	120	2^{80}
SPONGENT-160/320/160	11 8*	2^{56} 2^{28}	13	2^{79}	Ours	240	2^{80}
SPONGENT-224/224/16	-	-	8	2^{91}	Ours	120	2^{112}
SPONGENT-224/224/112	10	2^{60}	11	2^{62}	Ours	170	2^{112}
SPONGENT-224/448/224	10 8*	2^{47} 2^{21}	11	2^{57}	Ours	340	2^{112}
SPONGENT-256/256/16	-	-	11	2^{110}	Ours	140	2^{128}
SPONGENT-256/256/128	15	2^{79}	15	2^{75}	Ours	195	2^{128}
SPONGENT-256/512/256	15 9*	2^{73} 2^{28}	15	2^{73}	Ours	385	2^{128}

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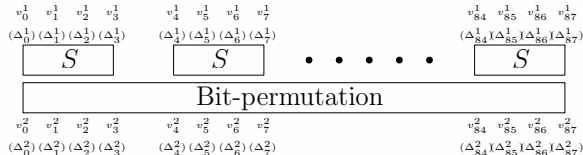
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■ Analyzing the dependency of input bits

- Employing Liu et al.'s model that describes the dependency between the internal difference and the value pair [LIM20]
- ✓ Using it twice: to reveal the dependency of input bits, and to enumerate the value pairs satisfying a differential characteristics



■ Procedure to enumerate the DoF (Consisting of 6 steps)



Step 1. Express this dependency as CNF by Espresso

$$(v_0^0 \vee v_0^1 \vee \Delta_0^0 \vee \neg \Delta_0^1) \wedge (v_2^0 \vee v_5^1 \neg \Delta_2^0 \vee \Delta_5^1) \wedge \cdots \wedge (v_{77}^{r-1} \vee v_{79}^r \vee \Delta_{77}^{r-1} \vee \Delta_{79}^{r-1}) = 1$$

Step 2. Assign a given differential characteristics to CNF generated in Step 1

$$(v_0^0 \vee v_0^1 \vee 1 \vee \neg 0) \wedge (v_2^0 \vee \neg v_5^1 \vee 0 \vee 0) \wedge \cdots \wedge (v_{77}^{r-1} \vee v_{79}^r \vee 0 \vee 1) = 1$$

Step 3. Express all Variables by the variable of input bits

$$(v_0^0 \vee v_3^0 \vee v_{18}^0) \wedge (v_2^0 \vee \neg v_5^0) \wedge \cdots \wedge (v_2^0 \vee v_{16}^0 \vee v_{35}^0 \vee \neg v_{55}^0 \vee v_{79}^0) = 1$$

Step 4. Convert the CNF into a product of terms, minimizing the number of terms, using simplify-logic tools (We use simplify-logic() in SymPy library for Python)

$$((v_0^0 \vee v_3^0) \oplus v_{18}^0) \wedge (v_2^0 \vee \neg v_5^0) \wedge \cdots \wedge ((v_2^0 \vee v_{16}^0) \oplus (v_{35}^0 \vee v_{79}^0)) = 1$$

■ Details of Step 5 (2 rounds of SPONGENT-160/160/80)

- Analyze all terms in the product of terms and categorize variables
- **Free bits**: A variable that does not appear in the Boolean function, such as v_1 , v_2 , and v_5
- **Fixed bits**: A variable that appears in a term consisting only of itself
- **Dependent bits**: A Variable that depends on other variables in a term

✓ Grouping dependent bits by how they depend on each other

$\neg(v_{70} \oplus v_{71} \oplus (v_{69} \wedge v_{72}) \oplus (v_{70} \wedge v_{71} \wedge v_{72})) \rightarrow$ Categorized into the same group

$$\begin{aligned}
 & v_{49} \wedge \neg v_{52} \wedge v_{57} \wedge \neg v_{60} \wedge v_{67} \wedge v_{75} \wedge (v_{65} \vee v_{68}) \wedge (v_{73} \vee v_{76}) \wedge (v_{50} \vee \neg v_{51}) \wedge (v_{51} \vee \neg v_{50}) \wedge (v_{58} \vee \neg v_{59}) \\
 & \wedge (v_{59} \vee \neg v_{58}) \wedge (v_{66} \vee \neg v_{65}) \wedge (v_{74} \vee \neg v_{73}) \wedge (\neg v_{66} \vee \neg v_{68}) \wedge (\neg v_{74} \vee \neg v_{76}) \wedge (v_{78} \oplus v_{79} \oplus (v_{77} \wedge v_{80}) \oplus (v_{78} \wedge v_{79} \wedge v_{80})) \\
 & \wedge (v_{61} \oplus (v_{61} \wedge v_{64}) \oplus (v_{62} \wedge v_{63}) \oplus (v_{62} \wedge v_{64}) \oplus (v_{63} \wedge v_{64}) \oplus (v_{62} \wedge v_{63} \wedge v_{64})) \wedge (v_{77} \oplus (v_{77} \wedge v_{80}) \oplus (v_{78} \wedge v_{79}) \oplus (v_{78} \wedge v_{80}) \\
 & \oplus (v_{79} \wedge v_{80}) \oplus (v_{78} \wedge v_{79} \wedge v_{80})) \wedge \neg(v_{53} \oplus (v_{53} \wedge v_{56}) \oplus (v_{54} \wedge v_{55}) \oplus (v_{54} \wedge v_{56}) \oplus (v_{55} \wedge v_{56}) \oplus (v_{54} \wedge v_{55} \wedge v_{56})) \\
 & \wedge \neg(v_{69} \oplus (v_{69} \wedge v_{72}) \oplus (v_{70} \wedge v_{71}) \oplus (v_{70} \wedge v_{72}) \oplus (v_{71} \wedge v_{72}) \oplus (v_{70} \wedge v_{71} \wedge v_{72})) \wedge ((v_{58} \wedge v_{59}) \vee \neg(v_{50} \wedge v_{51}) \vee (v_{61} \oplus (v_{61} \wedge v_{64}) \\
 & \oplus (v_{62} \wedge v_{63}) \oplus (v_{62} \wedge v_{64}) \oplus (v_{63} \wedge v_{64}) \oplus (v_{62} \wedge v_{63} \wedge v_{64}))) \wedge ((v_{74} \oplus (v_{73} \wedge v_{76}) \oplus (v_{74} \wedge v_{76})) \vee \neg(v_{66} \oplus (v_{65} \wedge v_{68}) \oplus (v_{66} \wedge v_{68})) \\
 & \vee (v_{78} \oplus v_{79} \oplus (v_{77} \wedge v_{80}) \oplus (v_{78} \wedge v_{79} \wedge v_{80}))) \wedge ((v_{65} \oplus v_{66} \oplus v_{68} \oplus (v_{65} \wedge v_{68})) \vee \neg(v_{73} \oplus v_{74} \oplus v_{76} \oplus (v_{73} \wedge v_{76})) \\
 & \vee (v_{77} \oplus (v_{77} \wedge v_{80}) \oplus (v_{78} \wedge v_{79}) \oplus (v_{78} \wedge v_{80}) \oplus (v_{79} \wedge v_{80}) \oplus (v_{78} \wedge v_{79} \wedge v_{80}))) \wedge \neg(v_{70} \oplus v_{71} \oplus (v_{69} \wedge v_{72}) \oplus (v_{70} \wedge v_{71} \wedge v_{72})) = 1
 \end{aligned}$$

■ Details of Step 5 (2 rounds of SPONGENT-160/160/80)

- Variables in each category can be determined independently
- ✓ Enumerate the number of assignments for variables in each group independently in Step 6

Category	Bits	Values
Free bits	48	$V_{25}, V_{13}, V_{28}, V_{46}, V_{23}, V_{22}, V_{19}, V_{37}, V_{17}, V_{11}, V_{30}, V_8, V_{41}, V_6, V_{32}, V_{43}, V_{45}, V_{16}, V_{21}, V_{47}, V_{31}, V_{14}, V_{34}, V_{36}, V_{18}, V_{33}, V_4, V_{38}, V_{48}, V_{29}, V_5, V_{26}, V_3, V_{20}, V_{27}, V_1, V_{39}, V_{15}, V_{40}, V_{12}, V_{24}, V_{44}, V_{42}, V_9, V_{35}, V_7, V_2, V_{10}$
Fixed bits	6	$V_{49}, \neg V_{52}, V_{57}, \neg V_{60}, V_{67}, V_{75}$
Group 1	4	$V_{53}, V_{56}, V_{54}, V_{55}$
Group 2	4	$V_{69}, V_{72}, V_{70}, V_{71}$
Group 3	8	$V_{59}, V_{50}, V_{64}, V_{63}, V_{51}, V_{58}, V_{61}, V_{62}$
Group 4	10	$V_{80}, V_{78}, V_{77}, V_{73}, V_{68}, V_{66}, V_{65}, V_{76}, V_{79}, V_{74}$

■ Details of Step 6

- Count the number of value pairs for each group independently
- Construct Liu et al.'s model using SAT and find one solution
 - ✓ If the solver returns "UNSAT", the given differential characteristic is invalid
- Enumerate the number of solutions for each group by solving the SAT model multiple times
 - ✓ Variables in other groups are fixed by the assignment found in the above procedure
 - ✓ **The number of solutions is equal to the available DoF**
- **The total DoF is the product of the number of solutions in each group**

$$DoF = 2^{\# \text{ free bits}} \times \underbrace{N_1}_{\# \text{ solutions in group 1}} \times \underbrace{N_2}_{\# \text{ solutions in group 2}} \times \cdots \times \underbrace{N_a}_{\# \text{ solutions in group } a}$$

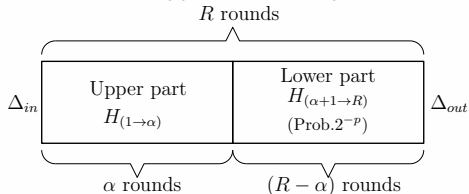
Contents

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3.1 Overview of Our Collision Attacks

■ Rebound-like attack

- Divide the underlying permutation into upper and lower parts



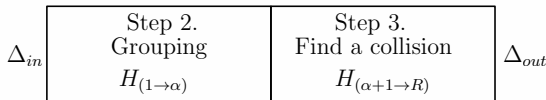
- For the upper part
 - ✓ Apply the Grouping method to obtain sufficient DoF
- For the lower part
 - ✓ Find a colliding pair using the DoF obtained in the upper part

- Complexity analysis

$$T = \underbrace{\frac{\alpha}{R} (2^{\#free\ bits} \times N_1 \times N_2 \times \cdots \times N_a)}_{\text{The upper part}} + \underbrace{\frac{R - \alpha}{R} \cdot 2^p}_{\text{The lower part}}$$

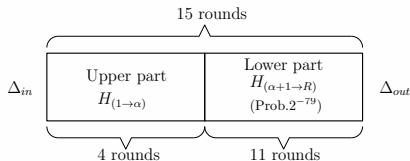
- Step 1: Search for a differential characteristic by SAT-based automatic search method
 - Restrict 2^P to be lower than the claimed security
- Step 2: Apply the Grouping method into the upper part
 - If the DoF obtained is smaller than 2^P , we repeat Step 1
- Step 3: Find a colliding pair
 - Explore a value pair satisfying a given differential characteristic in the lower part

Step 1. Search a differential characteristic ($\Delta_{in} \rightarrow \Delta_{out}$) with $\text{Prob.}(H_{(\alpha+1 \rightarrow R)})$



Application to the 15-round SPONGENT-256/256/128

Collision attacks on the 15-round SPONGENT-256/256/128

[illegible]

➤ At least 2^{-79} DoF is required in $H_{(1 \rightarrow 4)}$

Collision attacks on the 15-round SPONGENT-256/256/128

Category	Bits	Values
Free bits	64	$V_{13}, V_{23}, V_{12}, V_{43}, V_{53}, V_{60}, V_3, V_{34}, V_{37}, V_{14}, V_{11}, V_{30}, V_{19}, V_{49}, V_4, V_{25}, V_{59}, V_{64}, V_{18}, V_{22}, V_8, V_{24}, V_{51}, V_{58}, V_{21}, V_{16}, V_{56}, V_{17}, V_{39}, V_{33}, V_1, V_{15}, V_{32}, V_{28}, V_{55}, V_{62}, V_{52}, V_6, V_{50}, V_{42}, V_{54}, V_{10}, V_{38}, V_{48}, V_{45}, V_{46}, V_{26}, V_7, V_{47}, V_{40}, V_{44}, V_{31}, V_{35}, V_{27}, V_{29}, V_2, V_{63}, V_5, V_{57}, V_{36}, V_9, V_{20}, V_{41}, V_{61}$
Fixed bits	12	$V_{69}, V_{73}, V_{77}, V_{78}, \neg V_{79}, \neg V_{101}, \neg V_{103}, \neg V_{104}, \neg V_{106}, \neg V_{107}, V_{110}, \neg V_{112}$
Group 1	16	$V_{87}, V_{93}, V_{85}, V_{81}, V_{96}, V_{82}, V_{89}, V_{90}, V_{91}, V_{94}, V_{95}, V_{84}, V_{92}, V_{86}, V_{88}, V_{83}$
Group 2	36	$V_{100}, V_{65}, V_{67}, V_{68}, V_{105}, V_{75}, V_{115}, V_{123}, V_{66}, V_{119}, V_{114}, V_{74}, V_{99}, V_{109}, V_{124}, V_{71}, V_{120}, V_{122}, V_{127}, V_{108}, V_{102}, V_{111}, V_{116}, V_{128}, V_{72}, V_{80}, V_{117}, V_{97}, V_{113}, V_{118}, V_{125}, V_{70}, V_{76}, V_{121}, V_{98}, V_{126}$
Time of $H_{(1 \rightarrow 4)}$		$2^{77.1}$

- We obtain the DoF of 2^{64} , 2^7 , and 2^8 from free bits, Group 1, and Group 2, respectively.
- Time Complexity

$$T = \underbrace{\frac{4}{15} (2^{64} \times 2^7 \times 2^8)}_{H_{(1 \rightarrow 4)}} + \underbrace{\frac{11}{15} \cdot 2^{79}}_{H_{(5 \rightarrow 15)}} = 2^{79}$$

■ First third-party security analysis for collision resistance of SPONGENT

➤ * indicates that we found the real colliding pair

	Collision		Semi-free-start		Ref	Full round	Claimed security
	Rounds	Time	Rounds	Time			
SPONGENT-88/80/8	-	-	8*	2^{17}	Ours	45	2^{40}
	6†	$2^{55.2}$	-	-	[BKL ⁺ 11]		
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SPONGENT-160/160/80	11	2^{61}	14	2^{78}	Ours	120	2^{80}
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SPONGENT-256/512/256	15 9*	2^{73} 2^{28}	15	2^{73}	Ours	385	2^{128}

■ Summary

- We developed Liu et al.'s tool to obtain sufficient DoF
- We conducted the first collision attacks on the SPONGENT family
- We demonstrated real colliding pairs for several variants of SPONGENT

■ Future work

- Extend the Grouping method to cover more rounds
- Consider dependencies between the upper and lower parts

Thank you for your attention!

References I



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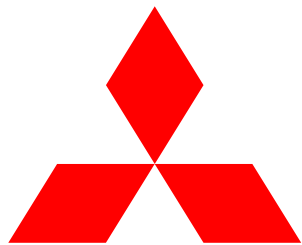
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